Profit Maximization for Online Advertising Demand-Side Platforms

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Outline

- Problem motivation and perspectives
- Optimization model preliminaries, assumptions, and properties
- Solution approach based on Lagrangian duality
- Synthetic computational results
- Conclusions and ongoing work
Problem Motivation
Problem Motivation

- What is a demand side platform? (DSP)

- DSPs *manage* the campaigns of many different advertisers and play a crucial role *connecting* them with publishers.
Problem Motivation cont.

- DSPs are faced with the challenge of managing advertisers’ campaigns by interacting with ad exchanges in a real time bidding environment.
  - Effective management requires forecasting the landscape of ad exchanges.

- We focus on campaign management, particularly how to balance:
  - Meeting advertisers’ goals and constraints
  - Profitability for the DSP

- DSPs may receive as many as a million ad requests per second and need to make decisions in real time.
  - Thus simple greedy heuristics are often employed.
Problem Formulation (in words)

- DSP profit maximization
- CPC/CPA pricing model
- Decision variables:
  - When a new impression arrives, who (among all the campaigns for the DSP) do we bid on behalf of and how much should we bid?
- Objective: maximize profit
- Constraints:
  - Campaign budget/pacing constraints
  - Targeting constraints
  - Supply (impression) availability constraints
Perspective and Contributions

- We develop a mathematical optimization formulation that:
  - Carefully models stochasticity in the real-time bidding process
  - Jointly optimizes over allocation strategies and bid prices
  - Accounts for limited supply of impression type inventory

- Our approach has several important features:
  - Scalability to the large-scale size of the problem
  - We address the stochastic nature of the problem
  - We account for the dynamic nature of the problem via model predictive control
A crucial input to our methodology is accurate forecasting of the value of an incoming impression, and how this value varies across different campaigns (e.g., CTR prediction).

**“DSP Analytics Pipeline”**

- Historical Data
- Statistical Model
- Decision Strategies

- CTR prediction
- Bid landscape modeling
- Impression arrival modeling
- Profit/goal optimization
- Budget pacing
- Ad quality optimization
Related Literature

- Revenue Management for the Publisher
  - [Balseiro et al. 2014] and also [B. Chen et al. 2014] study how publishers should optimally trade-off guaranteed contracts with RTB
  - [Y. Chen et al. 2011] studies how a publisher should optimally allocate impressions and set up bid prices for campaigns, under an implicit “central planner” assumption

- Revenue Management for the Ad Network
  - [Ciocan and Farias 2012] provides theoretical performance guarantees for a model predictive control approach

- Profit Optimization for the Advertiser
  - [Zhang et al. 2014] studies optimal RTB bidding for an advertiser (without impression allocation)

- Others…
Model Preliminaries
Model Preliminaries

- Planning over a fixed time horizon
- $\mathcal{I}$ is the set of impression types
- $\mathcal{K}$ is the set of campaigns
- Targeting constraints are specified via a bipartite graph

Diagram:

Impression Types

\[ |\mathcal{I}| \]

Campaigns

\[ |\mathcal{K}| \]
Impression types are defined via targeting (e.g. females, aged 25-34)

Each arrival of impression type $i$ corresponds to a real-time auction

For each impression type, we assume that we can use a bid landscape forecasting model:

- $\rho_i(b)$ is the probability of winning an auction for impression type $i$ when entering bid $b$
- $\beta_i^{\text{max}}(b)$ is the expected second price, i.e., the expected payment if we win, as a function of the bid

The total number of arrivals of impression type $i$ is a random variable with mean $s_i$
Modeling Campaigns

- Each campaign has a fixed budget $m_k$ over the time horizon.
  - Budget pacing can be incorporated by controlling this input.

- $\mathcal{I}_k$ is the set of impression types that campaign $k$ targets.
  - $\mathcal{K}_i$ is the set of campaigns targeted by impression type $i$.

- $q_k > 0$ is the amount that campaign $k$ is charged every time a click happens.

- $\theta_{ik}$ is the predicted CTR for users of impression type $i$ clicking on ads from campaign $k$.

- $r_{ik} := q_k \theta_{ik}$ is the expected cost per impression (eCPI) value, which is the expected amount of revenue the DSP earns each time an ad from campaign $k$ is shown to an impression $i$. 
Decision Variables and Corresponding Policy/Dynamics

- Decision variables:
  - $x_{ik}$ is the probability of choosing campaign $k$ to bid on behalf of when an arrival for impression type $i$ occurs
  - $b_{ik}$ is the corresponding bid price

Flip coins with probabilities $x_{ik}$ to decide which campaign to bid for
Policy Dynamics cont.

- Suppose that we bid $b_{ik}$ on behalf of campaign $k$

\[ \begin{align*}
\text{Win with probability } & \rho_i(b_{ik}) \\
\text{Click with probability } & \theta_{ik} \\
\text{Earn Revenue } & q_k > 0
\end{align*} \]
Deterministic optimization formulation, assuming all random variables take on their expected values:

\[
\begin{align*}
\text{maximize} & \quad \sum_{(i,k) \in \mathcal{E}} \left[ r_{ik} - \beta_i^\text{max}(b_{ik}) \right] s_i x_{ik} \rho_i(b_{ik}) \\
\text{subject to} & \quad \sum_{i \in \mathcal{I}_k} r_{ik} s_i x_{ik} \rho_i(b_{ik}) \leq m_k \quad \forall k \in \mathcal{K} \\
& \quad \sum_{k \in \mathcal{K}_i} x_{ik} \leq 1 \quad \forall i \in \mathcal{I} \\
& \quad x, b \geq 0.
\end{align*}
\]
Properties of the Deterministic Approximation

- Due to joint optimization over allocation probabilities and bid prices, the deterministic approximation is generally non-convex.

- “Difficulties” mainly arise due to the budget constraints.

- Without the budget constraints, it is optimal to bid truthfully, i.e., to set $b^*_ik = r_{ik}$ and to greedily choose campaigns.

- With budget constraints, it may be optimal for the DSP to underbid on a (relatively) less valuable impression due to the possibility of a more valuable impression arriving in the future.

- For fixed bid prices, solving for the optimal allocation is a linear optimization problem.
Solution Approach Based on Lagrangian Dual
Three Phase Solution Approach

- Phase 1: Solve (convex) dual problem obtained from Lagrangian relaxation of the deterministic problem
  - The main algorithm we use is subgradient descent (or some simple [e.g., stochastic] variant)

- Phase 2: Use optimal dual variables from Phase 1 to set bid prices

- Phase 3: Recover a “good” allocation strategy by solving the linear optimization problem obtained by fixing the bid prices determined from Phase 2
  - Solve using commercial LP solvers, or ADMM for large-scale problems
Useful Observations

\[
\begin{align*}
\text{maximize} & \quad \sum_{(i,k) \in \mathcal{E}} [r_{ik} - \beta_{i}^{\max}(b_{ik})]s_{i}x_{ik}\rho_{i}(b_{ik}) \\
\text{subject to} & \quad \sum_{i \in \mathcal{I}_{k}} r_{ik}s_{i}x_{ik}\rho_{i}(b_{ik}) \leq m_{k} \quad \forall k \in \mathcal{K} \\
& \quad x \in \mathcal{S} \quad (\mathcal{S} \text{ denotes supply constraints}) \\
& \quad x, b \geq 0.
\end{align*}
\]

- Phase 1 is based on the following (previous) observations:
  - The objective function is just the total expected profit in a second price auction
  - Without budget constraints, the optimal setting of bid prices is \( \tilde{b}_{ij} \), i.e., bidding truthfully
  - With budget constraints, it may be optimal to under bid – budget constraints are making the problem hard
Lagrangian Relaxation

- We put Lagrange multipliers $\lambda \in \mathbb{R}^m$ on the budget constraints and form the Lagrangian function:

$$L(x, b, \lambda) := \sum_{(i,k) \in \mathcal{E}} [r_{ik} - \beta_i^{\max}(b_{ik})] s_{i} x_{ik} \rho_i(b_{ik})$$

$$+ \sum_{k \in K} \lambda_k \left[ m_k - \sum_{i \in \mathcal{I}_k} r_{ik} s_i x_{ik} \rho_i(b_{ik}) \right]$$

- Moving the budget constraint to the objective makes the problem “easy”

- Lagrangian may be re-written as:

$$L(x, b, \lambda) = \sum_{(i,k) \in \mathcal{E}} [(1 - \lambda_k)r_{ik} - \beta_i^{\max}(b_{ik})] s_{i} x_{ik} \rho_i(b_{ik}) + \sum_{k \in K} \lambda_k m_k$$
Phase 1 – Dual Problem

\[ L(x, b, \lambda) = \sum_{(i,k) \in \mathcal{E}} [(1 - \lambda_k)r_{ik} - \beta_i^{\max}(b_{ik})]s_i x_{ik} \rho_i(b_{ik}) \]
\[ + \sum_{k \in \mathcal{K}} \lambda_k m_k \]

- The dual function is then \( L^*(\lambda) := \max_{x \in \mathcal{S}, b \geq 0} L(x, b, \lambda) \)

- The dual function is convex

- When computing the dual function, “it is optimal to bid truthfully and allocate impressions greedily”

- Thus the dual function and subgradients of the dual function may be computed efficiently

And the dual problem is:

\[ \min_{\lambda} L^*(\lambda) \]
\[ \text{subject to } 0 \leq \lambda_k \leq 1 \text{ for all } k \in \mathcal{K} . \]
Phase 1 – Dual Problem

\[ \min_{\lambda} L^*(\lambda) \]
subject to \( 0 \leq \lambda_k \leq 1 \) for all \( k \in K \).

- We use projected subgradient descent to solve this problem.
- Effective when the number of impressions and the number of campaigns are very large.
- This yields a vector of (approximately) optimal dual variables \( \lambda^* \).
- Importantly, \( L^*(\lambda^*) \) provides an upper bound on the optimal profit.
Phase 2 – Set Bid Prices

- Recall that the Lagrangian satisfies:

\[
L(x, b, \lambda) = \sum_{(i,k) \in \mathcal{E}} [(1 - \lambda_k) r_{ik} - \beta_i^{\text{max}}(b_{ik})] s_i x_{ik} \rho_i(b_{ik}) \\
+ \sum_{k \in \mathcal{K}} \lambda_k m_k
\]

- Thus, given optimal dual variables \( \lambda^* \), we interpret \((1 - \lambda_k^*) r_{ik}\) as a modified valuation/bid price that accounts for the budget constraint.

- In Phase 2, we set \( \hat{b}_{ik} := (1 - \lambda_k^*) r_{ik} \)
  - This setting is not necessarily optimal but should have good performance guarantees.
  - Proving good performance guarantees is ongoing.
Phase 3 – Allocation Recovery

- Phase 2 gives us a setting of the bid prices as \( \hat{b}_{ik} := (1 - \lambda^*_k) r_{ik} \)

- Phase 3: fix these bid prices and solve the deterministic linear optimization problem to recover allocation probabilities \( \hat{x} \)
  - We end up with an approximate solution \((\hat{x}, \hat{b})\) to the original problem

- Linear optimization solution approaches:
  - Commercial solvers – scales to moderate to large size problems
  - Distributed/parallel ADMM based on decomposition across campaigns and impression types – scales to huge size problems
Each impression type and each campaign line has a quality score ($QS_i$ and $QS_k$) that is uniformly distributed on $[0,1]$.

- # of campaigns targeting impression type $i$ is $\text{Bin}(m, QS_i)$.
- $\rho_i(\cdot)$ is a maximum of $\text{Bin}(k, QS_i)$ uniform RVs.
- The CTR is given by $\theta_{ik} := QS_i \cdot QS_k$.
- Campaigns pay $1$ for each click.
Synthetic Computational Set-up cont.

- We compare the policy implied by our approach to a simple greedy baseline policy.

- Simulations are based on our distributional assumptions (i.e., assuming perfect forecasting).

- Recall that campaigns pay $1 for each click.

- The baseline policy chooses the campaign $j$ with largest value of $\theta_{ik}$ and uses $\theta_{ik}$ as the corresponding bid price.
Synthetic Computational Results

- Example 1: 100 campaigns, 100 impression types; budget is constant across campaigns, forecasted supply is constant across impression types

- Our approach solved the joint (allocation, bid price) problem to within 13% of optimality

- Simulation Statistics (averaged over 500 runs):

<table>
<thead>
<tr>
<th>Relative Profit (our policy/baseline)</th>
<th>1.257</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative cost</td>
<td>0.286</td>
</tr>
<tr>
<td>Relative Revenue</td>
<td>0.759</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Our policy</th>
<th>Baseline policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budget utilization</td>
<td>0.483</td>
<td>0.636</td>
</tr>
<tr>
<td>Profit/Revenue</td>
<td>0.807</td>
<td>0.487</td>
</tr>
</tbody>
</table>
Synthetic Computational Results cont.

- Same example, but now campaign budget is correlated with quality score

<table>
<thead>
<tr>
<th>Relative Profit (our policy/baseline)</th>
<th>1.576</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative cost</td>
<td>0.431</td>
</tr>
<tr>
<td>Relative Revenue</td>
<td>0.677</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Our policy</th>
<th>Baseline policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budget utilization</td>
<td>0.542</td>
<td>0.801</td>
</tr>
<tr>
<td>Profit/Revenue</td>
<td>0.500</td>
<td>0.215</td>
</tr>
</tbody>
</table>
Example 2: 100 campaigns, 10 impression types

Here we examine the effect of varying the average budget of each campaign (results averaged over 500 runs)
Conclusions and Ongoing Work

- Developed a mathematical optimization formulation for the management of a DSP that balances profitability with meeting advertisers’ targeting goals and budget constraints
  - Our approach accounts for uncertainty in the real time bidding process
  - Jointly optimizes over allocation strategies and bid prices
- Developed a two phase solution approach to solve the non-convex joint (bid price, allocation) problem based on Lagrangian relaxation
Conclusions and Ongoing Work cont.

- Compared our policy against a baseline policy in simulations
  - Results indicate that our policy generates significantly more profit, mainly by reducing costs by avoiding overly-aggressive bidding strategies

- Ongoing work:
  - Extensions to incorporate advertisers’ utility functions, model predictive control, CPM and/or oCPC pricing, robustness to uncertainty in parameter estimation, …
  - Theoretically characterize the gap between Lagrangian relaxation upper bound and recovered primal solution
  - More extensive computational evaluations based on real-world data
Thank You!