

Designing Auctions for Search Ads

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With input from various Google Research Scientists and Engineers

Rich Ad Auctions

Old Search Ads

FTD® Fresh Flowers & Gifts | Up To 33% Off Fresh Flowers

[Ad](#) www.ftd.com/ ▼

Shop FTD® Today & Send Fresh Flowers & Gifts, Hand-Delivered By An FTD® Florist!

1-800-FLOWERS.COM® | Same Day Delivery Available

[Ad](#) www.1800flowers.com/ ▼

100% Satisfaction Guarantee! Shop Flowers & Gifts For Any Occasion.

\$10 Off Teleflora® Flowers | Same & Next-Day Delivery | teleflora.com

[Ad](#) www.teleflora.com/Flower/Delivery ▼

All Flowers Hand-Crafted & Delivered by Local Florists In High-Quality Vases!

\$19.99 - Same Day Flowers | Express Same Day Delivery

[Ad](#) www.fromyouflowers.com/ ▼

20% Off All Items - Same Day Delivery - Fast, Easy & Affordable. Highest Customer Satisfaction with...

New Search Ads

1-800-FLOWERS.COM® | Same Day Delivery Available

[Ad](#) www.1800flowers.com/flower/delivery ▼ [\(888\) 907-3196](tel:(888)907-3196)

Whatever the occasion, count on 1-800-Flowers® to deliver smiles. Shop flowers & gifts. Get Same Day Delivery on Our Wide Selection of Flowers And Gifts. Shop 1800FLOWERS®! Satisfaction Guarantee.

[Truly Original Gifts. Deliver to 190+ Countries. 2018 Stevie Silver Winner.](#)

[Roses - from \\$34.99 - Elegant Rose Arrangements · More ▼](#)

Deal of The Week

Check Out All of This Weeks Great Flower Deals. Don't Miss Out.

Same Day Delivery

Explore Our Selection Of Flowers Available For Same Day Delivery!

Bouqs® July 4th Sitewide Sale | Save 20% Off with code | bouqs.com

[Ad](#) www.bouqs.com/ ▼

[★★★★★ Rating for bouqs.com: 4.8 - 1,097 reviews](#)

Offer ends soon. Shop our farm-fresh selection of sunflowers, roses & more. Get free shipping on all orders \$100+. Fresh From Farm. Flat Pricing, No Upsells. Next/Same Day Available.

[Our Most Popular Bouqs · Shop All Bouqs · Start a Subscription](#)

[Same Day Delivery - from \\$60.00 - Order by 11am PST · More ▼](#)

Send \$19.99 Flowers | Save On All Flower Deliveries | SendFlowers.com

[Ad](#) www.sendflowers.com/ ▼

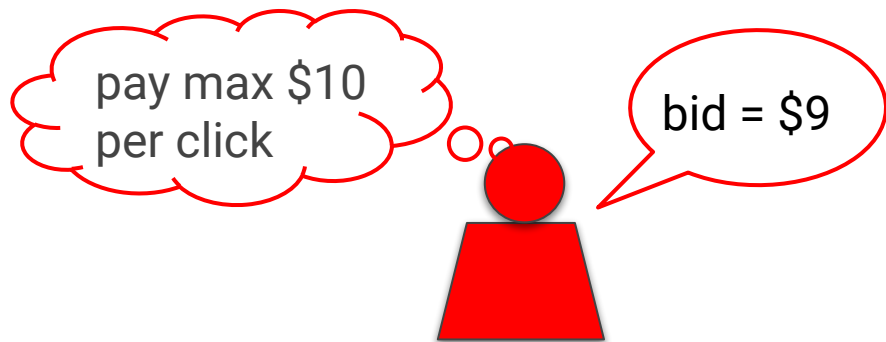
Order Now & Send Flowers Today. Delivery Today. Satisfaction Guaranteed. Sitewide Sale.

Auction Design

Allocation Rule: Algorithm to select ads

Payment Rule: Algorithm to compute payments (cost per click (CPC))

Advertiser Model



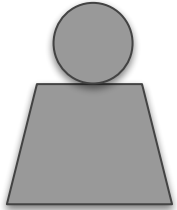
Value per click : maximum willingness to pay

Report: Bid per click

Goal: Maximize $utility = CTR * (value - CPC)$; CTR = expected number of clicks

Truthfulness: maximize utility with $bid = value$

Auctioneer Model



Auctioneer

Maximize economic efficiency: $\sum_{\text{Shown ads}} \text{value} * \text{CTR}$

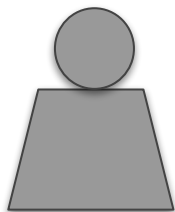
- Show ads from advertisers that value them the most

Outline

- Position Auctions
- Designing Rich Ad Auctions
- Optimal Rich Ad Auction
- Greedy Auction

Position Auction

Position Auction



Auctioneer

Choose ads for k positions

Allocation Rule: Assign ads to position in the $eCPM = bid * CTR$ order

Generalized Second Price (GSP) Payment Rule

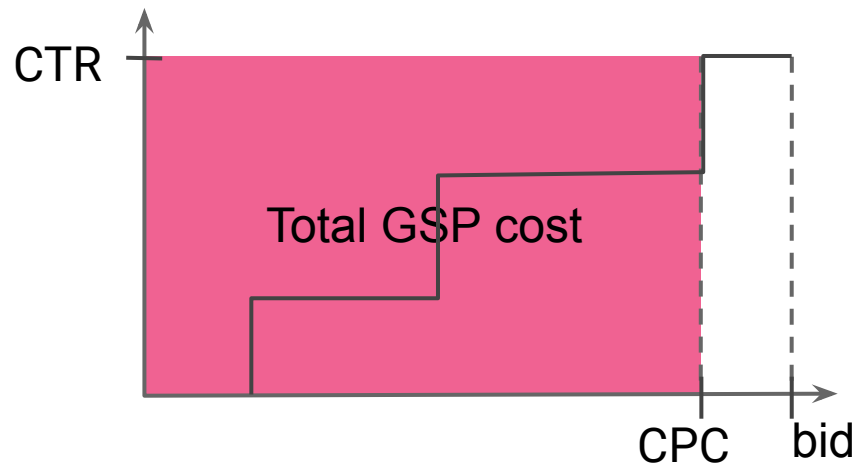
Payment Rule:

- Minimum threshold below which the ad loses clicks

$$\text{CPC} = \text{next-eCPM} / \text{CTR}$$

Where $\text{eCPM} = \text{bid} * \text{CTR}$

- Same price charged for all clicks



From second price to GSP

GSP generalizes celebrated second price [Vickrey'61] auction for single item

Second price auction in single position is **truthful** - optimal to bid true value independent of other's bid

Simple generalization to multiple positions not truthful!

From second price to GSP

GSP generalizes celebrated second price [Vickrey'61] auction for single item

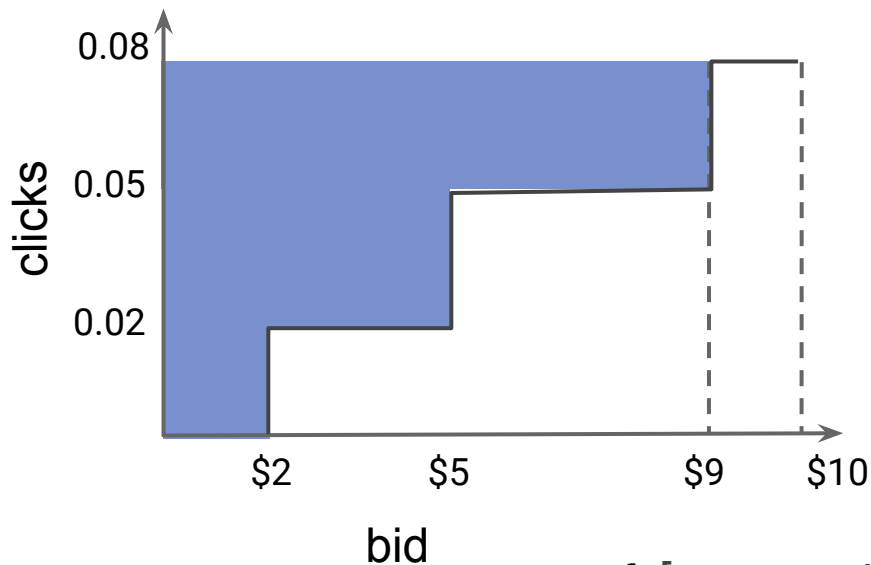
Second price auction in single position is **truthful** - optimal to bid true value independent of other's bid

Simple generalization to multiple positions not truthful!

Takeaway #1: Auctions that work for single item may break when extended to multiple items

Vickrey-Clarke-Groves (VCG) payment rule

Payment Rule: Charge for each incremental clicks the minimum bid at which the clicks are obtained



Payment

$$= \$2 * 0.02 + \$5 * (0.05 - 0.02) + \\ \$9 * (0.08 - 0.05) = 0.46$$

$$\text{CPC} = \text{payment/clicks} = \$5.75$$

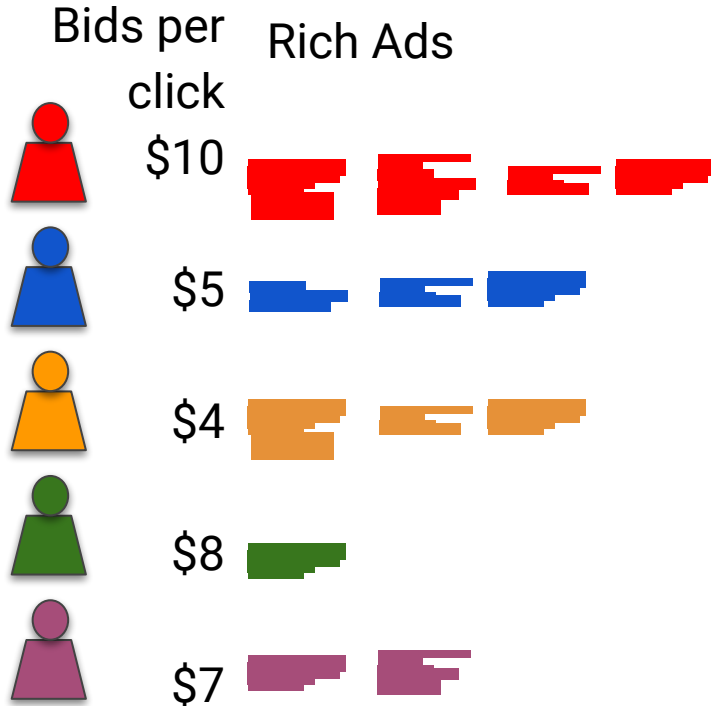
Ref: [Aggarwal et al. 2006]

Designing Rich Ad Auctions

Outline

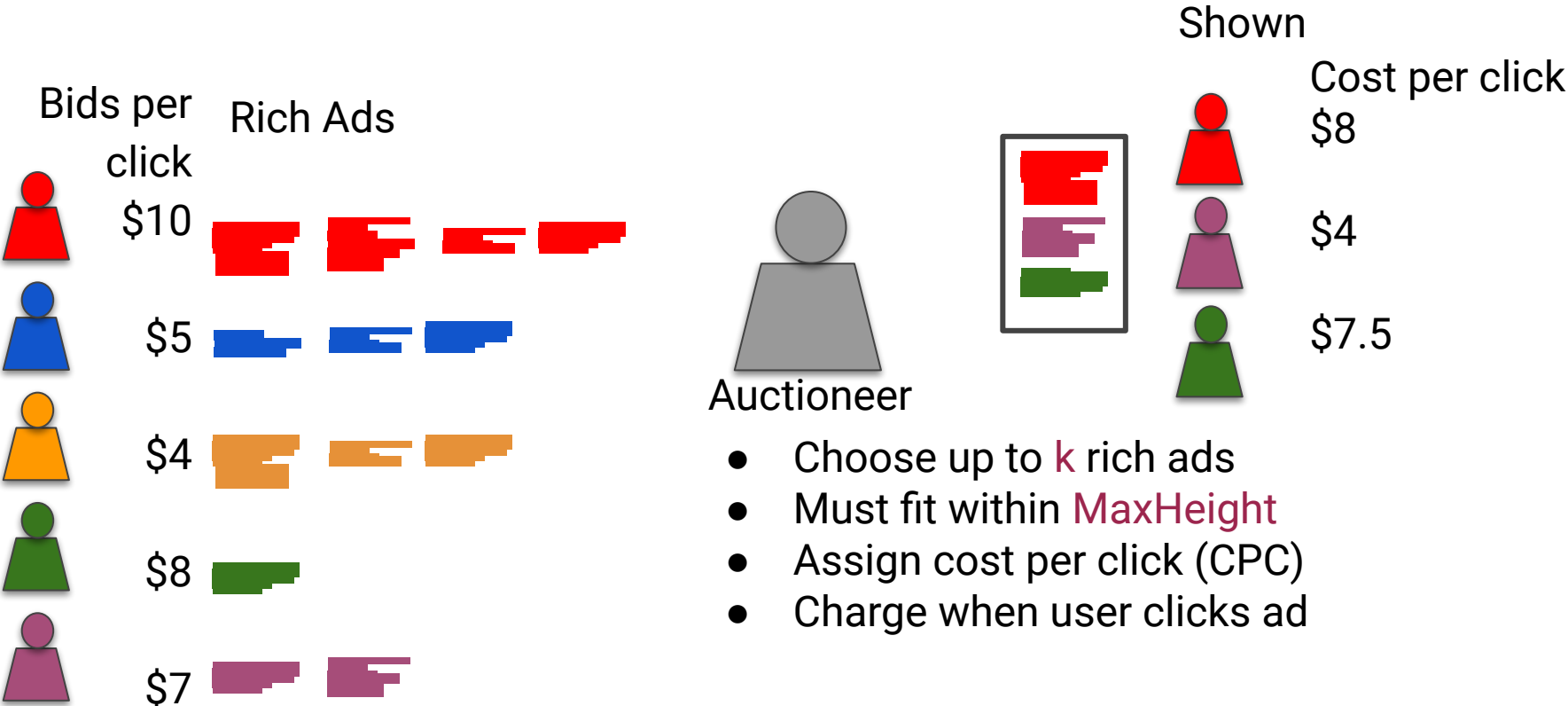
- Position Auctions
- Designing Rich Ad Auctions
- Optimal Rich Ad Auction
- Greedy Auction

Rich Ad Auctions



- Bid
 - Maximum price for a click
 - Same for all rich ads
- Rich ads differ in
 - Height in pixels
 - Information provided
 - Click Through Rate (CTR)

Rich Ad Auctions



Truthful Rich Ad Auction

Truthful: Optimal to report true value independent of what others bid

Preferable when starting from scratch

Why truthful?

- Ease of bidding
- Easier to extend

Truthful Rich Ad Auction

Truthful: Optimal to report true value independent of what others bid

Preferable when starting from scratch

Why truthful?

- -
- Takeaway #2: Consider implementing truthful auctions

Building on GSP

GSP used for more than a decade...

- Well established
- Understood and optimized for by advertisers, engineers
- Steady state bids optimized for GSP
- Very challenging to switch auction to VCG [Varian, Harris 2013]

Goal: Generalize GSP for Rich Ads

Rich ad auction should have

- Same allocation and payment as GSP when unconstrained
- **Bid monotonicity:** Get same or more clicks if bidding higher
- **Second pricing principle:** charge minimum threshold to lose clicks

Strategizing about Rich Ads

Advertisers can be strategic about which rich ads they provide.

Rich ad truthfulness:

Optimal to provide all rich ads

Rich ad monotonicity:

Advertiser should not get more more clicks by opting out of rich ads

Strategizing about Rich Ads

Advertisers can be strategic about which rich ads they provide.

Rich ad truthfulness:

Optimal to provide all rich ads

Rich ad monotonicity:

Adv **Takeaway #3: Beware of different ways participants can be strategic**

Optimal Rich Ad Auction

Outline

- Position Auctions
- Designing Rich Ad Auctions
- Optimal Rich Ad Auction
- Simple Greedy Auction

Optimal Allocation

Allocation Rule: Choose up to k rich ads, only one per advertiser to

$$\text{Maximize } \sum_{\text{Selected ads}} \text{eCPM} = \sum_{\text{Selected ads}} \text{bid} * \text{CTR}$$

Optimal Allocation: Computational Challenge

Allocation Rule: Choose up to k rich ads, only one per advertiser to

$$\text{Maximize } \sum_{\text{Selected ads}} \text{eCPM} = \sum_{\text{Selected ads}} \text{bid} * \text{CTR}$$

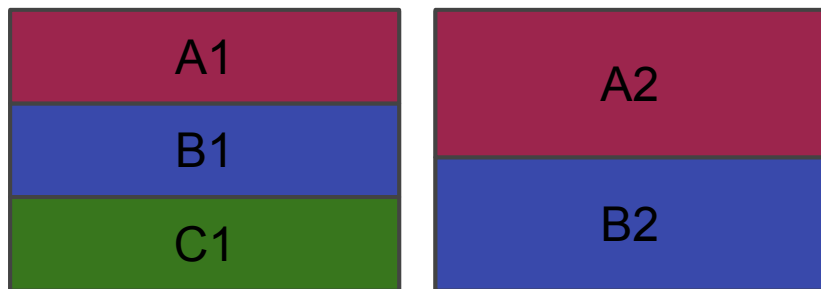
Computational Challenge:

- Knapsack problem: Find best packing of rich ads with Max-Height
- Greedy not optimal, implement dynamic program or brute force
- Pushing real world latency limits

Optimal allocation is not rich ad monotone

Optimal allocation trades off space between advertisers

Example:



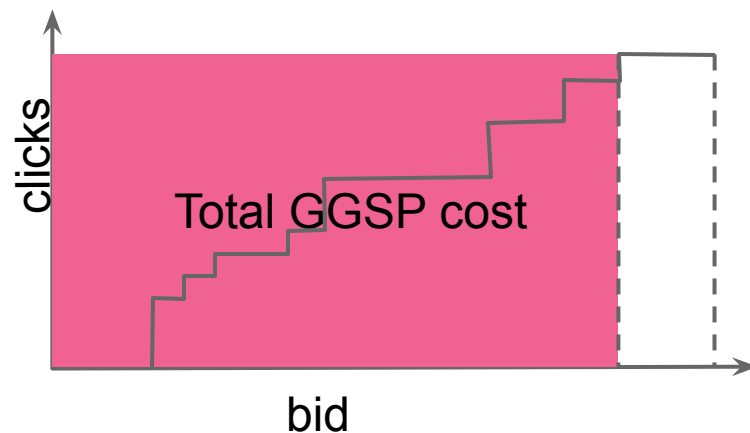
- Config on left is best
- A, B get more clicks in config on right.
- A or B can opt-out of smaller rich ad to ensure config on the right wins.

GSP like payment rule

Payment Rule: Generalized GSP (GGSP)

- Minimum threshold at which lose clicks
- Same CPC for all clicks

[Muthukrishnan'09, Cavallo et al.'17]

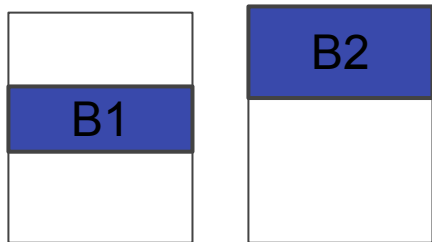


GGSP is a bit more complex

GGSP price = $\max_{\text{losing configs}} \text{min. Bid to beat config}$

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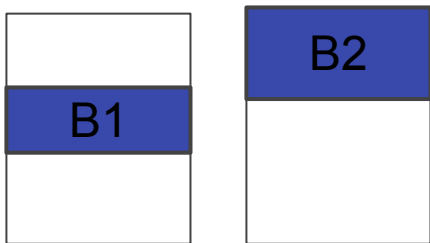
GGSP price = $\max_{\text{losing configs}} \min. \text{ Bid to beat config}$



- Advertiser appears in both configs
- Lowering bids lowers sum-eCPM of both
- Price = bid where the scores become equal.

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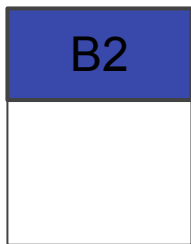
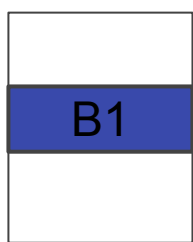


- Advertiser appears in both configs
- Lowering bids lowers sum-eCPM of both
- Price = bid where the scores become equal.

$$\text{CPC}(i) = \frac{\text{sum-ecpm}(\text{losing config without } i) - \text{sum-ecpm}(\text{winning config without } i)}{\text{CTR}(i, \text{winning-config}) - \text{CTR}(i, \text{losing config})}$$

GGSP is a bit more complex

GGSP price = $\max_{\text{losing configs}} \min. \text{ Bid to beat config}$



No longer just pay the bid of the next ad

- Price = bid where the scores become equal.

$$\text{CPC}(i) = \frac{\text{sum-ecpm}(\text{losing config without } i) - \text{sum-ecpm}(\text{winning config without } i)}{\text{CTR}(i, \text{winning-config}) - \text{CTR}(i, \text{losing config})}$$

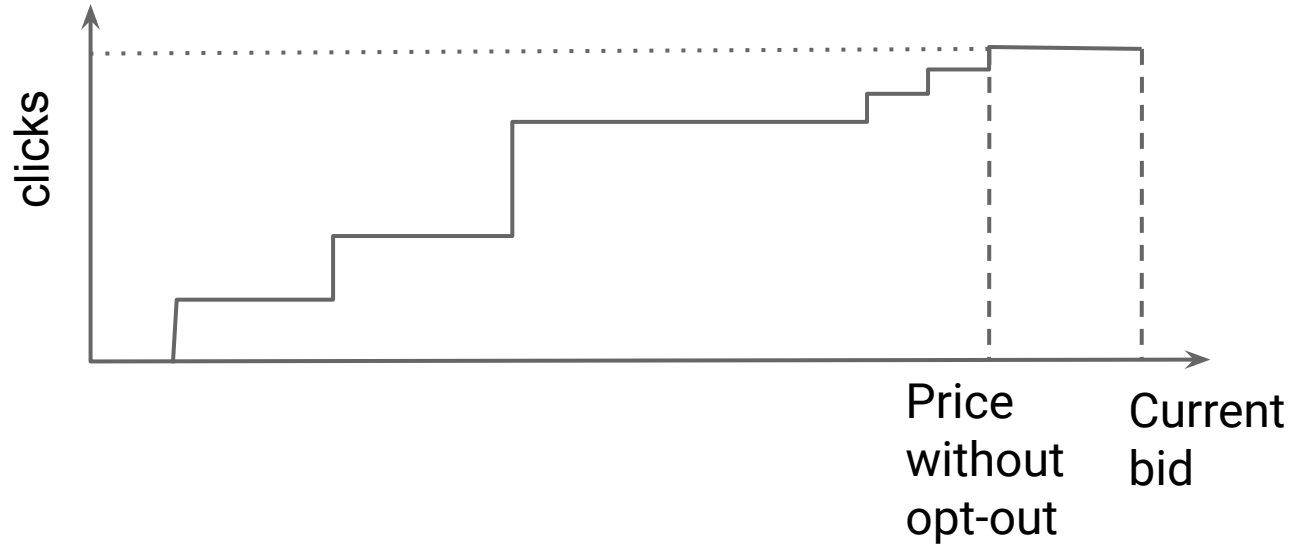
Properties of Generalized GSP

- + Same prices as GSP in special cases
- Large increase in CPC for a small increase in clicks

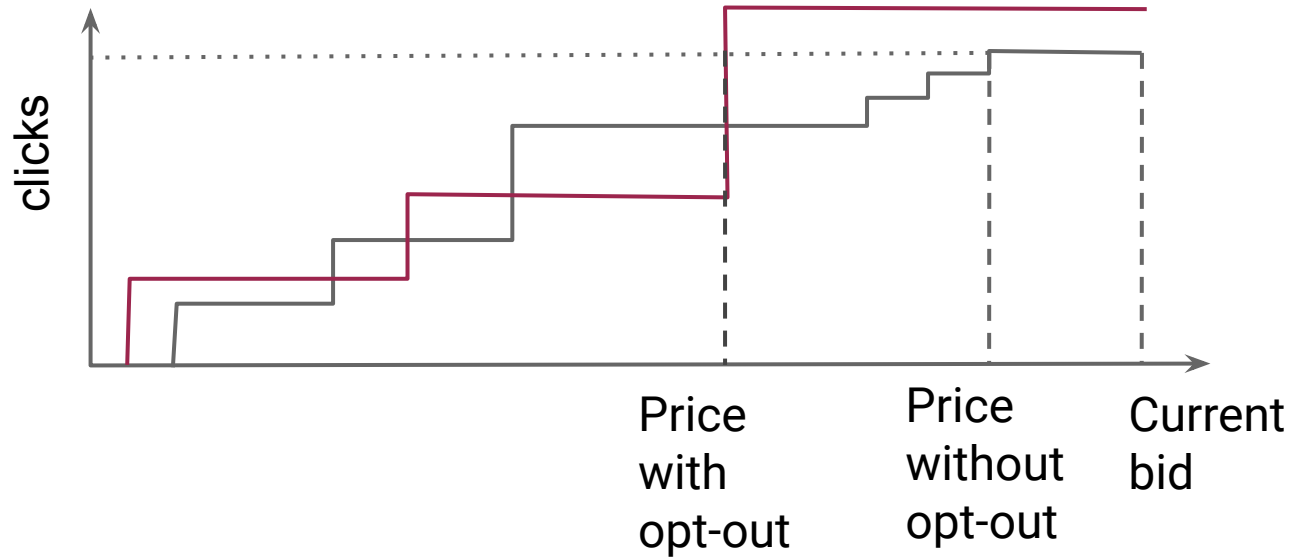


Takeaway #3: Generalizations of second price do not retain all the nice properties

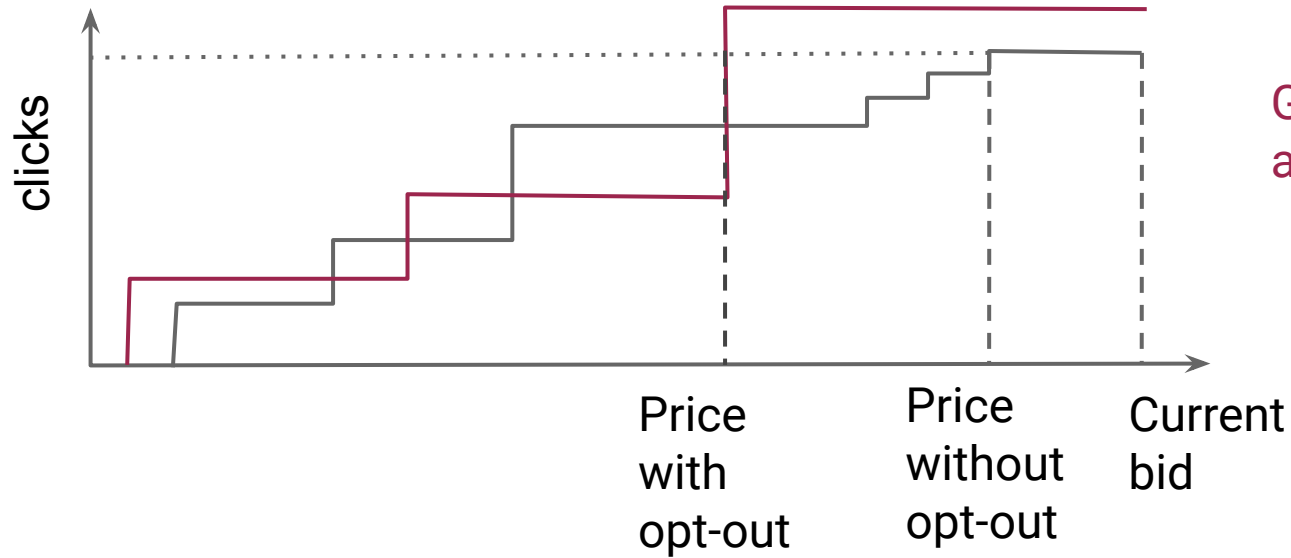
Lack of rich ad monotonicity breaks GGSP



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Get more clicks
at lower price

Truthful payment rule for Rich Ad Auctions

Vickrey(1961), Clarke(1971), Groves(1973) provide general truthful auction

Allocation rule finds the optimal allocation

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Payment rule:

For each shown ad i , charge damage caused to others.

$$\text{Payment}(i) = \left(\begin{array}{c} \text{best efficiency} \\ \text{without } i \end{array} \right) - \left(\begin{array}{c} \text{Efficiency of ads other} \\ \text{than } i \text{ in selection} \end{array} \right)$$

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Truthful in both bids and rich ads

Computationally expensive!

Greedy Auction

Outline

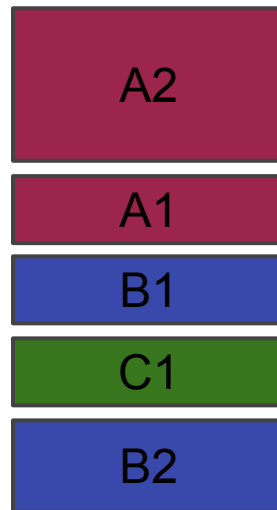
- Position Auctions
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Greedy Auction

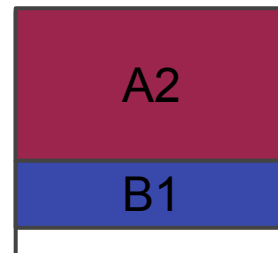
Allocation Rule:

- Pick ads in $eCPM = bid * CTR$ order
- Only one rich ad per advertiser
- Stop when space runs out

eCPM order



Allocation



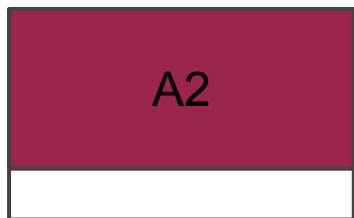
Greedy Allocation Rule: Properties

- + Bid monotone: bidding higher gets more clicks
- + Rich ad monotone: always show best rich ad for each advertiser
- + Efficient when space is not a constraint

Greedy Allocation Rule: Properties

- + Bid monotone: bidding higher gets more clicks
- + Rich ad monotone: always show best rich ad for each advertiser
- + Efficient when space is not a constraint
- Inefficient when space is constraint

Greedy Outcome



Optimal Outcome



Generalized Second Price (GSP) Payment Rule

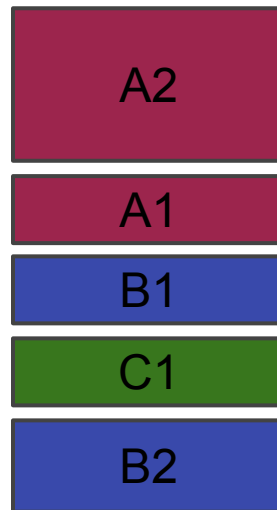
Payment Rule:

$$\text{CPC} = \frac{\text{eCPM of next ad by competitor}}{\text{CTR}}$$

Where eCPM = bid * CTR

- Minimum threshold below which the ad loses clicks
- Same price charged for all clicks

eCPM order



Approximate VCG like pricing does not work!

VCG like Payment rule:

For each shown ad i ,

$$\text{Payment}(i) = \left(\begin{array}{l} \text{sum-eCPM of} \\ \text{output of greedy} \\ \text{auction without } i \end{array} \right) - \left(\begin{array}{l} \text{sum-eCPM in the selected} \\ \text{allocation of ads other} \\ \text{than } i \end{array} \right)$$

Approximate VCG like pricing does not work!

VCG like Payment rule:

For each shown ad i ,

$$\text{Payment}(i) = \left(\begin{array}{c} \text{sum-eCPM of} \\ \text{output of SGA} \\ \text{without } i \end{array} \right) - \left(\begin{array}{c} \text{sum-eCPM in the selected} \\ \text{allocation of ads other} \\ \text{than } i \end{array} \right)$$

This mechanism is not truthful!

Proof of truthfulness relies on solving optimization problem optimally.

Approximate VCG like pricing does not work!

VCG like Payment rule:

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$$\text{Payment}(i) = \left(\begin{array}{c} \text{sum-eCPM of} \\ \text{output of SGA} \\ \text{without } i \end{array} \right) - \left(\begin{array}{c} \text{sum-eCPM in the selected} \\ \text{allocation of ads other} \\ \text{than } i \end{array} \right)$$

Takeaway #4: VCG paired with approximation algorithms is not truthful

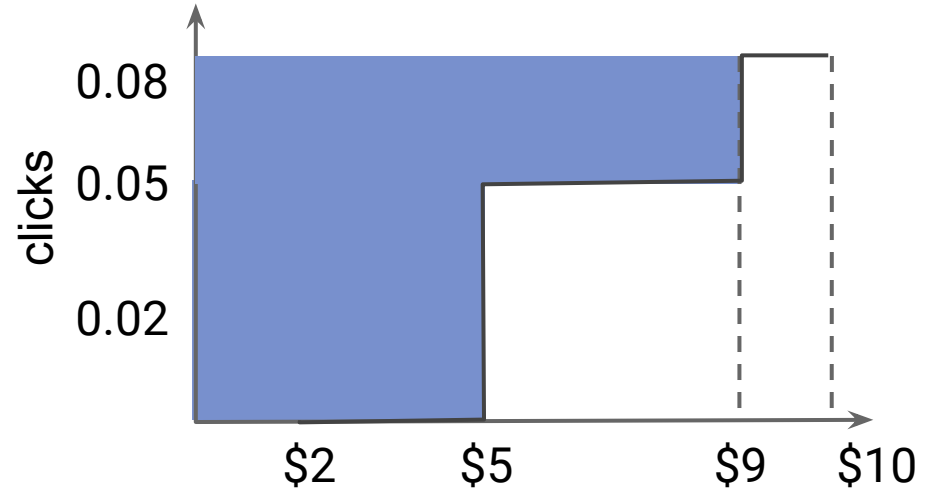
This

Proc

Truthful Greedy Auction

Truthful pricing rule [Myerson' 81]

- Construct the bid vs clicks curve
- Charge for each incremental clicks the minimum bid at which the clicks are obtained

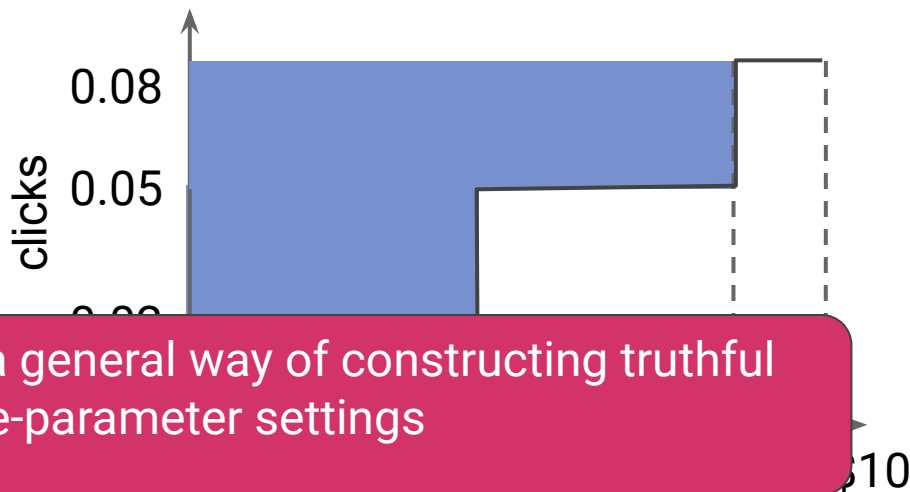


bid

Truthful Greedy Auction

Truthful pricing rule [Myerson' 81]

- Construct the bid vs clicks curve
- Charge for each incremental clicks the minimum bid at which the



Takeaway #5: Myerson provides a general way of constructing truthful auctions in single-parameter settings

Summary

	OPT	Greedy eCPM
Efficiency	Optimal	Optimal if space is not tight
Bid mon.	Yes	Yes
Rich ad mon.	No	Yes
GSP pricing	GGSP	Same as GSP
Truthful pricing	VCG	Myerson's pricing

Takeaways for Auction Design

1. Auctions for single items may break when extended to multiple items
2. Consider implementing truthful auctions
3. Beware of different ways participants can be strategic
4. Generalizations of second price do not retain all the nice properties
5. VCG with approximation algorithms not truthful
6. Myerson provides a general way of constructing truthful auctions



Thank You!

Revenue maximization

- Configuration Auctions with VCG or GGSP pricing can have low revenue
- Also not revenue monotone - more advertisers, higher bids can lead to lower revenue.

[Hartline et al. 2018] core auctions to obtain higher revenue, not truthful, require solving the Optimal allocation $O(n \log n)$ times.

Open Question: tractable revenue optimizing auctions.

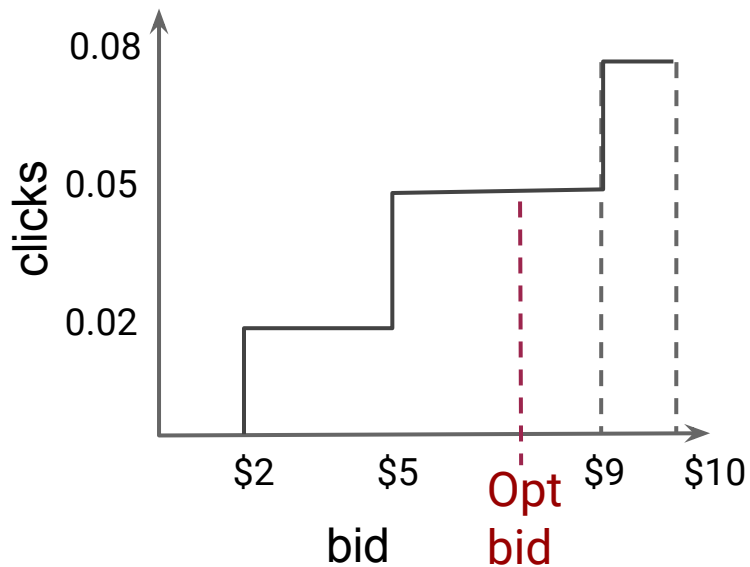
[Cavallo et al. 2017] heuristic

- Builds on the Greedy knapsack heuristic.
- Local search to improve the quality of the solution.
- Can be paired with VCG or GSP pricing

Doesn't have good incentive properties.

- Not bid or rich ad monotone
- Doesn't evaluate optimal solution, VCG won't be truthful

GSP is not truthful



Value = \$10



Bid	Clicks	CPC	Utility
Bid \geq \$9	0.08	\$9	0.08
\$9 > bid \geq \$5	0.05	\$5	0.25
\$5 > bid \geq \$2	0.02	\$2	0.16

$$\text{Utility} = \text{CTR} * (\text{value} - \text{CPC})$$

Truthfulness of VCG: Proof Sketch

Truthful: Optimal to report true value independent of other's bid

Utility = clicks * value - payment = Area under the curve

