Designing Auctions for Search Ads

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Joint work with Gagan Aggarwal, Aranyak Mehta
With input from various Google Research Scientists and Engineers
Rich Ad Auctions

Old Search Ads

1-800-FLOWERS.COM® | Same Day Delivery Available

100% Satisfaction Guaranteed! Shop Flowers & Gifts For Any Occasion.

$10 Off Teleflora® Flowers | Same & Next-Day Delivery | teleflora.com

All Flowers Hand-Crafted & Delivered by Local Florists in High-Quality Vases!

$19.99 - Same Day Flowers | Express Same Day Delivery

20% Off All Items - Same Day Delivery - Fast, Easy & Affordable. Highest Customer Satisfaction with...

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Truly Original Gifts. Deliver to 190+ Countries. 2018 Stevie Silver Winner.

Roses - from $34.99 - Elegant Rose Arrangements - More

Deal of The Week

Check Out All of This Weeks Great Flower Deals. Don't Miss Out.

Same Day Delivery

Explore Our Selection Of Flowers Available For Same Day Delivery!

Bouqs® July 4th Sitewide Sale | Save 20% Off with code | bouqs.com

★★★★★ Rating for bouqs.com: 4.8 - 1,097 reviews


Our Most Popular Bouqs - Shop All Bouqs - Start a Subscription

Same Day Delivery - from $60.00 - Order by 11am PST - More

Send $19.99 Flowers | Save On All Flower Deliveries | SendFlowers.com

Auction Design

**Allocation Rule:** Algorithm to select ads

**Payment Rule:** Algorithm to compute payments (cost per click (CPC))
Advertiser Model

**Goal:** Maximize utility = CTR * (value - CPC); CTR = expected number of clicks

**Truthfulness:** maximize utility with bid = value

Value per click: maximum willingness to pay

Report: Bid per click

pay max $10 per click

bid = $9
Auctioneer Model

Maximize economic efficiency: \[ \sum_{\text{Shown ads}} \text{value} \times \text{CTR} \]

- Show ads from advertisers that value them the most
Outline

- Position Auctions
- Designing Rich Ad Auctions
- Optimal Rich Ad Auction
- Greedy Auction
Position Auction
Position Auction

Choose ads for k positions

**Allocation Rule:** Assign ads to position in the \( \text{eCPM} = \text{bid} \times \text{CTR} \) order
Generalized Second Price (GSP) Payment Rule

Payment Rule:

- Minimum threshold below which the ad loses clicks
  \[
  \text{CPC} = \frac{\text{next-eCPM}}{\text{CTR}}
  \]
  Where eCPM = bid * CTR

- Same price charged for all clicks
From second price to GSP

GSP generalizes celebrated second price [Vickrey'61] auction for single item.

Second price auction in single position is **truthful** - optimal to bid true value independent of other's bid.

Simple generalization to multiple positions not truthful!
From second price to GSP

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Second price auction in single position is **truthful** - optimal to bid true value independent of other's bid

Simple generalization to multiple positions not truthful!

Takeaway #1: Auctions that work for single item may break when extended to multiple items
Vickrey-Clarke-Groves (VCG) payment rule

**Payment Rule:** Charge for each incremental clicks the minimum bid at which the clicks are obtained

![Graph showing payment rule]

**Payment**

\[
\text{Payment} = 2 \times 0.02 + 5 \times (0.05 - 0.02) + 9 \times (0.08 - 0.05) = 0.46
\]

**CPC**

\[
\text{CPC} = \frac{\text{payment}}{\text{clicks}} = 5.75
\]

Ref: [Aggarwal et al. 2006]
Designing Rich Ad Auctions

Outline

- Position Auctions
- Designing Rich Ad Auctions
- Optimal Rich Ad Auction
- Greedy Auction
Rich Ad Auctions

- **Bid**
  - Maximum price for a click
  - Same for all rich ads

- **Rich ads differ in**
  - Height in pixels
  - Information provided
  - Click Through Rate (CTR)

<table>
<thead>
<tr>
<th>Bids per click</th>
<th>Rich Ads</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10</td>
<td><img src="image" alt="Red Ads" /></td>
</tr>
<tr>
<td>$5</td>
<td><img src="image" alt="Blue Ads" /></td>
</tr>
<tr>
<td>$4</td>
<td><img src="image" alt="Yellow Ads" /></td>
</tr>
<tr>
<td>$8</td>
<td><img src="image" alt="Green Ads" /></td>
</tr>
<tr>
<td>$7</td>
<td><img src="image" alt="Purple Ads" /></td>
</tr>
</tbody>
</table>
Rich Ad Auctions

- Choose up to $k$ rich ads
- Must fit within MaxHeight
- Assign cost per click (CPC)
- Charge when user clicks ad

**Bids per click**: $10, $5, $4, $8, $7

**Rich Ads**:
- Red
- Blue
- Orange
- Green
- Purple

**Cost per click**:
- $8
- $4
- $7.5

**Shown**
- Cost per click
- Rich Ads
Truthful Rich Ad Auction

**Truthful**: Optimal to report true value independent of what others bid

Preferable when starting from scratch

**Why truthful?**

- Ease of bidding
- Easier to extend
Truthful Rich Ad Auction

**Truthful:** Optimal to report true value independent of what others bid

Preferable when starting from scratch

**Why truthful?**

- Takeaway #2: Consider implementing truthful auctions
Building on GSP

GSP used for more than a decade...

- Well established
- Understood and optimized for by advertisers, engineers
- Steady state bids optimized for GSP
- Very challenging to switch auction to VCG [Varian, Harris 2013]
Goal: Generalize GSP for Rich Ads

Rich ad auction should have

- Same allocation and payment as GSP when unconstrained
- **Bid monotonicity:** Get same or more clicks if bidding higher
- **Second pricing principle:** charge minimum threshold to lose clicks
Strategizing about Rich Ads

Advertisers can be strategic about which rich ads they provide.

**Rich ad truthfulness:**
Optimal to provide all rich ads

**Rich ad monotonicity:**
Advertiser should not get more clicks by opting out of rich ads
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**Takeaway #3:** Beware of different ways participants can be strategic.
Optimal Rich Ad Auction

Outline

- Position Auctions
- Designing Rich Ad Auctions
- Optimal Rich Ad Auction
- Simple Greedy Auction
Optimal Allocation

**Allocation Rule:** Choose up to k rich ads, only one per advertiser to

\[ \text{Maximize} \quad \sum_{\text{Selected ads}} \text{eCPM} = \sum_{\text{Selected ads}} \text{bid} \times \text{CTR} \]
Optimal Allocation: Computational Challenge

**Allocation Rule:** Choose up to $k$ rich ads, only one per advertiser to

\[
\text{Maximize } \sum_{\text{Selected ads}} \ e\text{CPM} = \sum_{\text{Selected ads}} \ \text{bid} \times \text{CTR}
\]

**Computational Challenge:**
- Knapsack problem: Find best packing of rich ads with Max-Height
- Greedy not optimal, implement dynamic program or brute force
- Pushing real world latency limits
Optimal allocation is not rich ad monotone

Optimal allocation trades off space between advertisers

Example:

- Config on left is best
- A, B get more clicks in config on right.
- A or B can opt-out of smaller rich ad to ensure config on the right wins.
GSP like payment rule

**Payment Rule:** Generalized GSP (GGSP)

- Minimum threshold at which lose clicks
- Same CPC for all clicks

[Muthukrishnan'09, Cavallo et al.'17]
GGSP is a bit more complex

\[ \text{GGSP price} = \max_{\text{losing configs}} \min \text{ Bid to beat config} \]
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GGSP price = \[
\max_{\text{losing configs}} \ \min \text{. Bid to beat config}
\]

- Advertiser appears in both configs
- Lowering bids lowers sum-eCPM of both
- Price = bid where the scores become equal.
GGSP is a bit more complex

GGSP price = \( \max_\text{losing configs} \min \text{Bid to beat config} \)

- Advertiser appears in both configs
- Lowering bids lowers sum-eCPM of both
- Price = bid where the scores become equal.

\[
CPC(i) = \frac{\text{sum-ecpm}(\text{losing config without i}) - \text{sum-ecpm}(\text{winning config without i})}{\text{CTR}(i, \text{winning-config}) - \text{CTR}(i, \text{losing config})}
\]
GGSP is a bit more complex

- Advertiser appears in both configs
- Lowering bids lowers sum-eCPM of both
- Price = bid where the scores become equal.

\[
\text{CPC}(i) = \text{sum-ecpm}(\text{losing config without i}) - \text{sum-ecpm}(\text{winning config without i})
\]

\[
\text{CTR}(i, \text{winning-config}) - \text{CTR}(i, \text{losing config})
\]

GGSP price = \(\max_{\text{losing configs}} \min_{\text{Bid to beat config}}\)
Properties of Generalized GSP

➕ Same prices as GSP in special cases

➖ Large increase in CPC for a small increase in clicks

Takeaway #3: Generalizations of second price do not retain all the nice properties
Lack of rich ad monotonicity breaks GGSP
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Get more clicks at lower price
Truthful payment rule for Rich Ad Auctions

Vickrey (1961), Clarke (1971), Groves (1973) provide general truthful auction

Allocation rule finds the optimal allocation
Truthful payment rule for Rich Ad Auctions

Vickrey(1961), Clarke(1971), Groves(1973) provide general truthful auction

Allocation rule finds the optimal allocation

**Payment rule:**
For each shown ad $i$, charge damage caused to others.

$$\text{Payment}(i) = \left( \text{best efficiency without } i \right) - \left( \text{Efficiency of ads other than } i \text{ in selection} \right)$$
Truthful payment rule for Rich Ad Auctions

Vickrey (1961), Clarke (1971), Groves (1973) provide general truthful auction

Allocation rule finds the optimal allocation

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$$\text{Payment}(i) = \left( \frac{\text{best efficiency without } i}{\text{Efficiency of ads other than } i \text{ in selection}} \right)$$

*Truthful in both bids and rich ads*

*Computationally expensive!*
Greedy Auction

Outline

● Position Auctions
● Designing Rich Ad Auctions
● Optimal Rich Ad Auction
● Greedy Auction
Greedy Auction

**Allocation Rule:**

- Pick ads in $\text{eCPM} = \text{bid} \times \text{CTR}$ order
- Only one rich ad per advertiser
- Stop when space runs out
Greedy Allocation Rule: Properties

+ Bid monotone: bidding higher gets more clicks
+ Rich ad monotone: always show best rich ad for each advertiser
+ Efficient when space is not a constraint
Greedy Allocation Rule: Properties

+ Bid monotone: bidding higher gets more clicks
+ Rich ad monotone: always show best rich ad for each advertiser
+ Efficient when space is not a constraint
- Inefficient when space is constraint

Greedy Outcome

Optimal Outcome

\[
\begin{array}{c}
\text{C1} \\
\text{B1} \\
\text{A1}
\end{array}
\]
Generalized Second Price (GSP) Payment Rule

Payment Rule:

\[ \text{CPC} = \frac{(\text{eCPM of next ad by competitor}) \times \text{CTR}}{\text{CTR}} \]

Where eCPM = bid * CTR

- Minimum threshold below which the ad loses clicks
- Same price charged for all clicks

![Diagram showing eCPM order]

- A2
- A1
- B1
- C1
- B2
Approximate VCG like pricing does not work!

**VCG like Payment rule:**
For each shown ad i,

\[
\text{Payment}(i) = (\text{sum-eCPM of output of greedy auction without } i) - (\text{sum-eCPM in the selected allocation of ads other than } i)
\]
Approximate VCG like pricing does not work!

VCG like Payment rule:
For each shown ad i,
Payment(i) = (sum-eCPM of output of SGA without i) - (sum-eCPM in the selected allocation of ads other than i)

This mechanism is not truthful!
Proof of truthfulness relies on solving optimization problem optimally.
Approximate VCG like pricing does not work!

VCG like Payment rule:
For each shown ad i,
\[
\text{Payment}(i) = \left( \text{sum-eCPM of output of SGA without i} \right) - \left( \text{sum-eCPM in the selected allocation of ads other than i} \right)
\]

Takeaway #4: VCG paired with approximation algorithms is not truthful
Truthful Greedy Auction

**Truthful pricing rule [Myerson' 81]**

- Construct the bid vs clicks curve
- Charge for each incremental clicks the minimum bid at which the clicks are obtained
Truthful Greedy Auction

Truthful pricing rule [Myerson' 81]

- Construct the bid vs clicks curve
- Charge for each incremental clicks the minimum bid at which the

Takeaway #5: Myerson provides a general way of constructing truthful auctions in single-parameter settings
# Summary

<table>
<thead>
<tr>
<th></th>
<th>OPT</th>
<th>Greedy eCPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency</td>
<td>Optimal</td>
<td>Optimal if space is not tight</td>
</tr>
<tr>
<td>Bid mon.</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Rich ad mon.</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>GSP pricing</td>
<td>GGSP</td>
<td>Same as GSP</td>
</tr>
<tr>
<td>Truthful pricing</td>
<td>VCG</td>
<td>Myerson's pricing</td>
</tr>
</tbody>
</table>
Takeaways for Auction Design

1. Auctions for single items may break when extended to multiple items
2. Consider implementing truthful auctions
3. Beware of different ways participants can be strategic
4. Generalizations of second price do not retain all the nice properties
5. VCG with approximation algorithms not truthful
6. Myerson provides a general way of constructing truthful auctions
Thank You!
Revenue maximization

- Configuration Auctions with VCG or GGSP pricing can have low revenue
- Also not revenue monotone - more advertisers, higher bids can lead to lower revenue.

[Hartline et al. 2018] core auctions to obtain higher revenue, not truthful, require solving the Optimal allocation $O(n \log n)$ times.

Open Question: tractable revenue optimizing auctions.
[Cavallo et al. 2017] heuristic

- Builds on the Greedy knapsack heuristic.
- Local search to improve the quality of the solution.
- Can be paired with VCG or GSP pricing

Doesn't have good incentive properties.
- Not bid or rich ad monotone
- Doesn't evaluate optimal solution, VCG won't be truthful
GSP is not truthful

Value = $10

<table>
<thead>
<tr>
<th>Bid</th>
<th>Clicks</th>
<th>CPC</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid ≥ $9</td>
<td>0.08</td>
<td>$9</td>
<td>0.08</td>
</tr>
<tr>
<td>$9 &gt; bid ≥ $5</td>
<td>0.05</td>
<td>$5</td>
<td>0.25</td>
</tr>
<tr>
<td>$5 &gt; bid ≥ $2</td>
<td>0.02</td>
<td>$2</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Utility = CTR * (value - CPC)
Truthfulness of VCG: Proof Sketch

Truthful: Optimal to report true value independent of other's bid

Utility = clicks * value - payment = Area under the curve

Bidding true value

Under Bidding

Over bidding

Negative area