

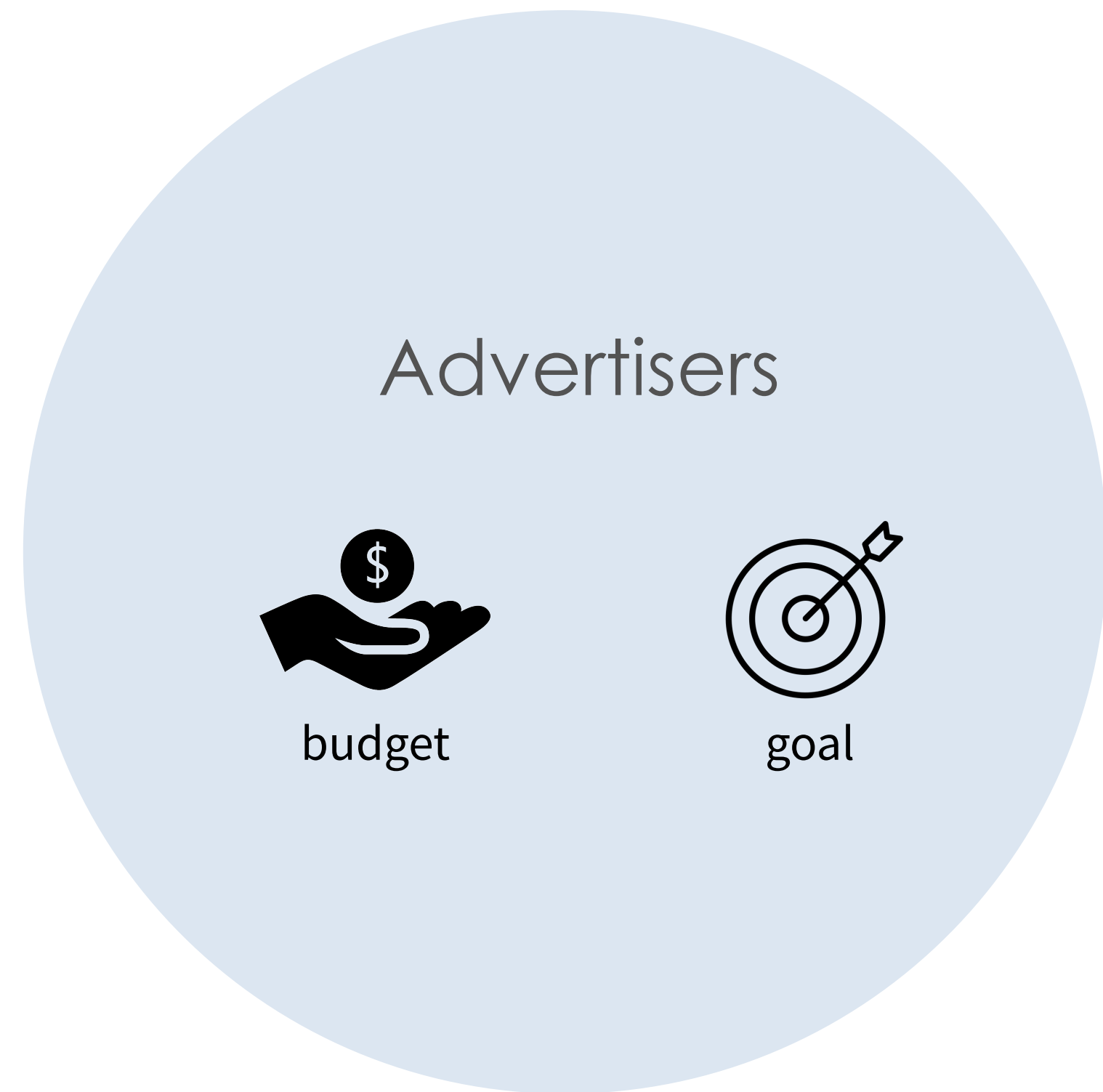
Bidding Agent Design in the LinkedIn Ad Marketplace

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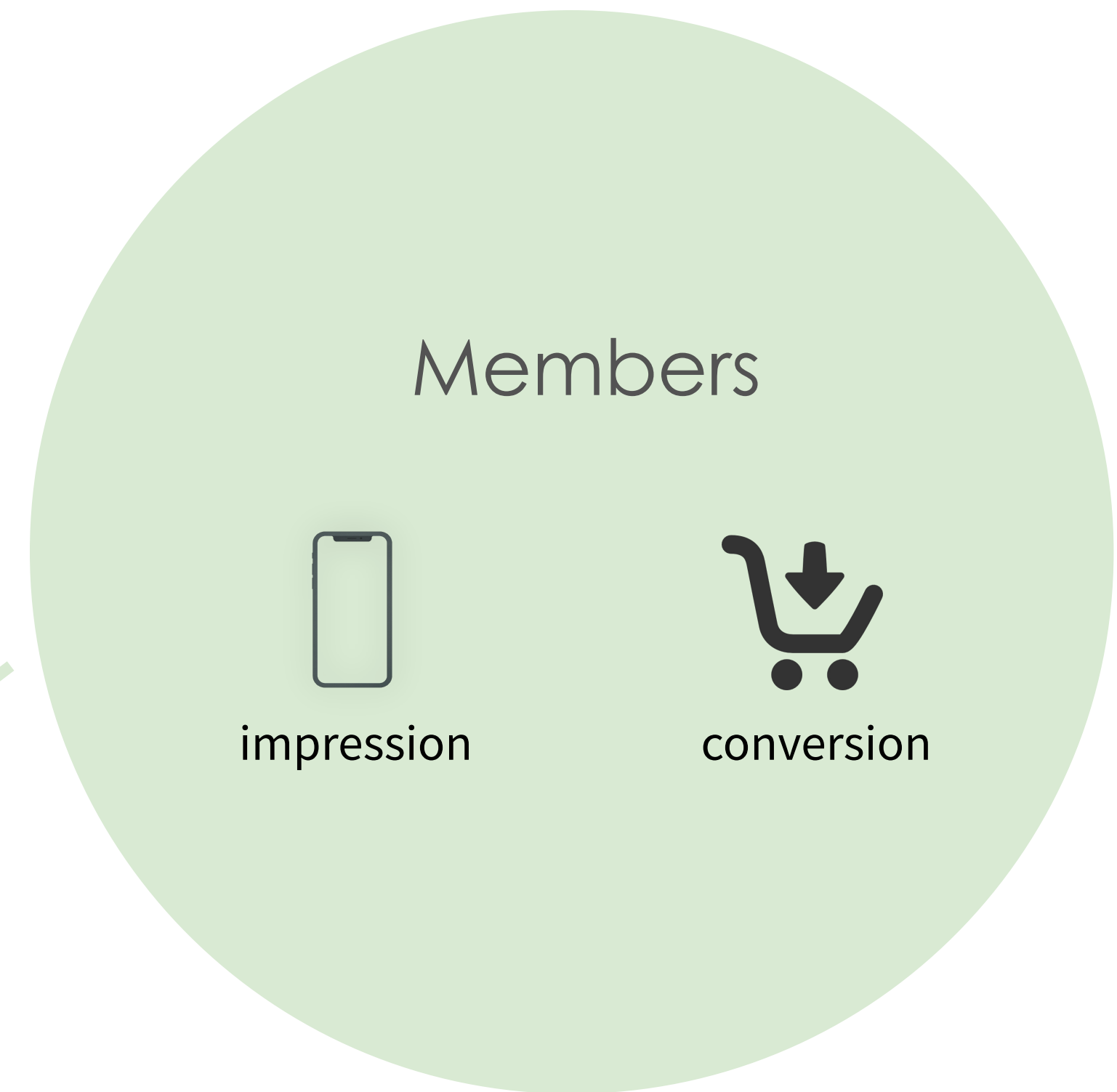


LinkedIn Ad Marketplace

Demand



Supply

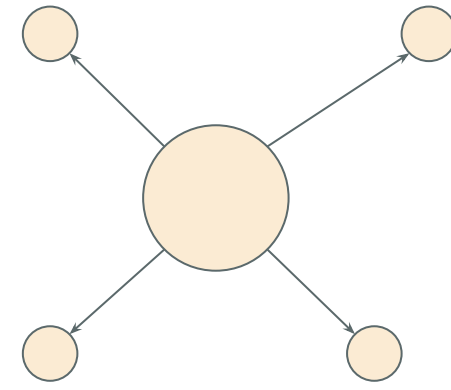


Auction



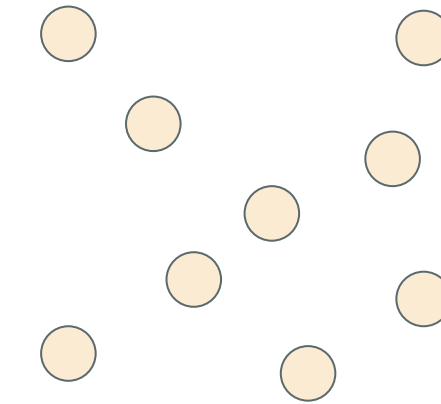
Design Philosophy

Centralized



v.s.

Decentralized



Formulate the Optimization Problem from Seller-defined Objective

One Giant Optimization Problem

"Online Resource Allocations"

Mehta et al. (2007); Abrams et al. (2008); Feldman et al. (2010); Chen et al. (2011); Grigas et al. (2017); Aggarwal et al. (2019); Balseiro et al. (2020);

Bidding strategy as a by-product of global optimization

Formulate the Optimization Problem from Individual Buyer's Objective

Massive Small Optimization Problems

Market Equilibrium as a by-product of bidding strategy

General Formulation

impression opportunities

value for the t-th impression

budget

$$\max_{b_t} \sum_{t=1}^T v_t G_t(b_t), \quad s.t. \quad \sum_{t=1}^T H_t(b_t) \leq B.$$

probability of winning

expected cost

Knapsack Problem: Modeling G_t and H_t as **step** functions [Chakrabarty et al. (2008); Zhou and Naroditskiy (2008)]

Fluid Approximation: Modeling G_t and H_t as **smooth** functions [Gallego and Van Ryzin (1994); Gummadi et al. (2013); Balseiro et al. (2015); Fernandez-Tapia et al. (2017)]

Surplus Maximization and Optimal Bidding Strategy

$$L(b_t, \lambda) = \sum_{t=1}^T [v_t G_t(b_t) - \lambda H_t(b_t)] + \lambda B.$$

Optimization in b_t amounts to **surplus** maximization:

$$\boxed{(v_t/\lambda)G_t(b_t) - H_t(b_t)}$$



budget adjusted value

First order optimality condition implies

$$b_t^*(\lambda) = \left(\frac{h_t}{g_t} \right)^{-1} \left(\frac{v_t}{\lambda} \right)$$

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$$b_t^*(\lambda) = \left(\frac{h_t}{g_t}\right)^{-1} \left(\frac{v_t}{\lambda}\right) \left\{ \begin{array}{l} \frac{v_t}{\lambda} \quad \text{2nd price auction} \\ \left(I + \frac{G_t}{g_t}\right)^{-1} \left(\frac{v_t}{\lambda}\right) \quad \text{1st price auction} \end{array} \right.$$

Online Methods in the Dual

KKT condition implies

$$\sum_{t=1}^T \boxed{H_t(b_t^*(\lambda^*))} = B$$

spend

The online loss function

$$L(b_t^*, \lambda) = \sum_{t=1}^T \boxed{L_t(b_t^*, \lambda) := [v_t G_t(b_t^*) - \lambda H_t(b_t^*)] + \lambda B/T.}$$

Derivative of the online loss

$$L'_t(b_t^*, \lambda) = \boxed{B/T - H_t(b_t^*)}$$

discrepancy between forecasted
and actual spend

Follow-the-leader: Find $\lambda_{t+1} = \operatorname{argmin}_{\lambda \geq 0} \sum_{\tau=1}^t L_\tau(b_\tau^*, \lambda)$.

Follow-the-regularized-leader: *Online Gradient Descent, Online Mirror Descent, etc.*

Other Types of Constraints

Cost Control (ROI target)

$$\max_{b_t} \sum_{t=1}^T v_t G_t(b_t), \quad s.t. \sum_{t=1}^T H_t(b_t) \leq \min \left(B, C \sum_{t=1}^T v_t G_t(b_t) \right).$$
$$b_t^*(\lambda, \mu) = \left(\frac{h_t}{g_t} \right)^{-1} \left(\frac{1 + \mu C}{\lambda + \mu} \cdot v_t \right)$$

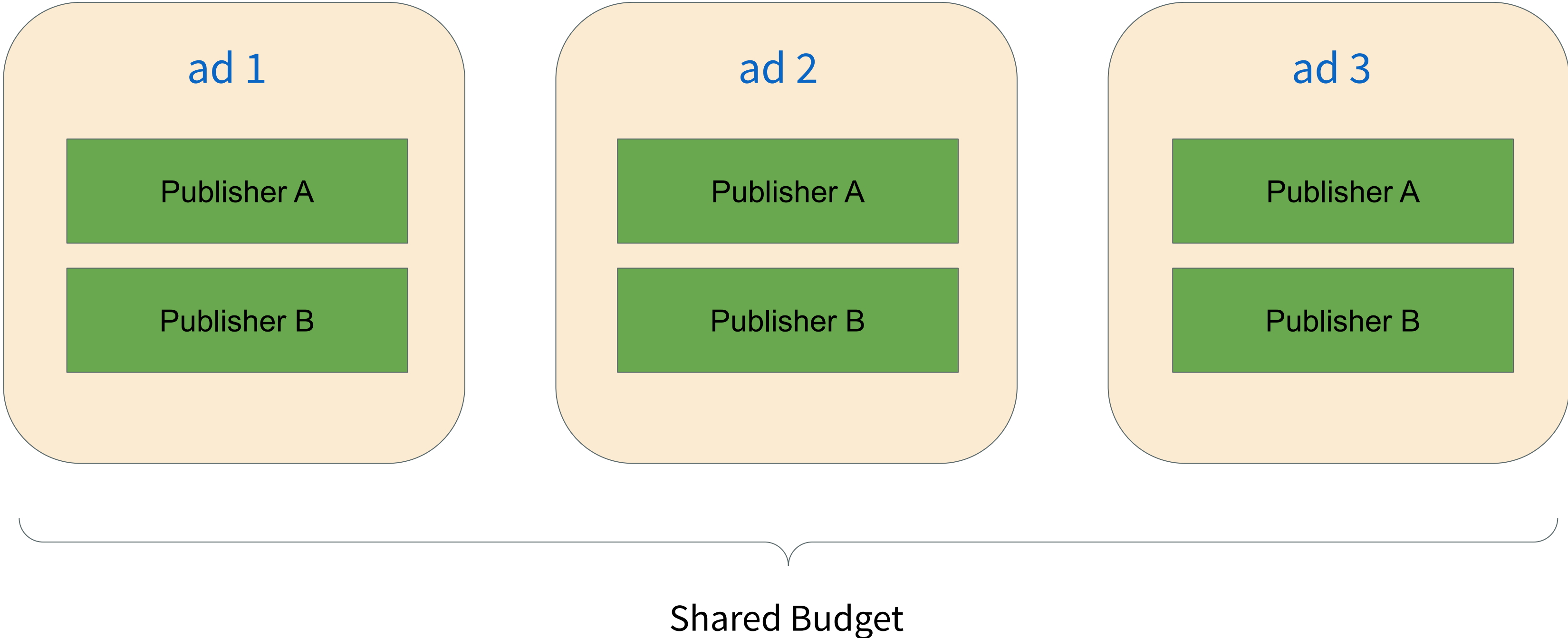
Budget Delivery Control

$$\max_{b_t} \sum_{t=1}^T v_t G_t(b_t), \quad s.t. \sum_{t=1}^T H_t(b_t) \leq B, \sum_{t \in T_k} H_t(b_t) \leq B_k, \forall k.$$
$$b_t^*(\lambda, \lambda_k) = \left(\frac{h_t}{g_t} \right)^{-1} \left(\frac{v_t}{\lambda + \lambda_k} \right)$$

Guaranteed Delivery

$$\max_{b_t} \sum_{t=1}^T v_t G_t(b_t), \quad s.t. \sum_{t=1}^T H_t(b_t) \leq B, \sum_{t \in T_k} v_t G_t(b_t) \geq V_k, \forall k.$$
$$b_t^*(\lambda, \mu_k) = \left(\frac{h_t}{g_t} \right)^{-1} \left(\frac{1 + \mu_k}{\lambda} \cdot v_t \right)$$

Multiple Placements, Group of Ads and Marginal ROI



**The proposed bidding framework ensures optimality, which implies
equality of marginal ROI among all ads across all placements**

Engineering Considerations

- Batch online update



- Parameter Tuning

- Normalization of λ
- Empirical vs. Asymptotic Regret

- Forecast Error

- Sensitivity Analysis on T
- Relative Forecast vs. Total Forecast

- Model Predictive Control

Initialization

Theorem (Gao, Yang, Chen, Liu, El Karoui '22)

Competitor's bids $\sim \text{Lognormal}(\mu, \sigma^2)$

value $\sim \text{Lognormal}(\mu', (\sigma')^2)$

Then optimal λ^* is the solution to

$$e^{\mu + \frac{\sigma^2}{2}} \Phi \left(\frac{\mu' - \mu - \ln \lambda^* - \sigma^2}{\sqrt{(\sigma')^2 + \sigma^2}} \right) = \frac{B}{T}$$

Thank you!