Bidding Agent Design in the LinkedIn Ad Marketplace

Yuan Gao (joint work with Kaiyu Yang, Yuanlong Chen, Min Liu and Noureddine El Karoui)
LinkedIn Ad Marketplace

Demand

Advertisers
- budget
- goal

Supply

Members
- impression
- conversion

Auction
Design Philosophy

Centralized

Formulate the Optimization Problem from Seller-defined Objective

One Giant Optimization Problem

*Bidding strategy as a by-product of global optimization*

Mehta et al. (2007); Abrams et al. (2008); Feldman et al. (2010); Chen et al. (2011); Grigas et al. (2017); Aggarwal et al. (2019); Balseiro et al. (2020);

Decentralized

Formulate the Optimization Problem from Individual Buyer's Objective

Massive Small Optimization Problems

*Market Equilibrium as a by-product of bidding strategy*
General Formulation

\[
\max_{b_t} \sum_{t=1}^{T} v_t G_t(b_t), \quad \text{s.t.} \quad \sum_{t=1}^{T} H_t(b_t) \leq B.
\]

**Impression opportunities**

**Value for the t-th impression**

**Budget**

**Probability of winning**

**Expected cost**

Knapsack Problem: Modeling \( G_t \) and \( H_t \) as *step* functions \[\text{Chakrabarty et al. (2008); Zhou and Naroditskiy (2008)}\]

Fluid Approximation: Modeling \( G_t \) and \( H_t \) as *smooth* functions \[\text{Gallego and Van Ryzin (1994); Gummadi et al. (2013); Balseiro et al. (2015); Fernandez-Tapia et al. (2017)}\]
Surplus Maximization and Optimal Bidding Strategy

\[ L(b_t, \lambda) = \sum_{t=1}^{T} [v_t G_t(b_t) - \lambda H_t(b_t)] + \lambda B. \]

Optimization in \( b_t \) amounts to \textit{surplus maximization}:

\[
\frac{(v_t / \lambda)}{G_t(b_t) - H_t(b_t)}
\]

\textit{budget adjusted value}

First order optimality condition implies

\[
b_t^*(\lambda) = \left( \frac{h_t}{g_t} \right)^{-1} \left( \frac{v_t}{\lambda} \right)
\]
Surplus Maximization and Optimal Bidding Strategy

\[ L(b_t, \lambda) = \sum_{t=1}^{T} \left[ v_t G_t(b_t) - \lambda H_t(b_t) \right] + \lambda B. \]

Optimization in \( b_t \) amounts to surplus maximization:

\[
\frac{(v_t/\lambda)G_t(b_t) - H_t(b_t)}{\text{budget adjusted value}}
\]

First order optimality condition implies

\[
b_t^*(\lambda) = \left( \frac{h_t}{g_t} \right)^{-1} \left( \frac{v_t}{\lambda} \right)
\]

\[
\frac{v_t}{\lambda}
\]

\[
\frac{v_t}{\lambda}
\]

\[
\left( \frac{I + G_t}{g_t} \right)^{-1} \left( \frac{v_t}{\lambda} \right)
\]

\[
1st \text{ price auction}
\]

\[
2nd \text{ price auction}
\]
Online Methods in the Dual

KKT condition implies

\[ \sum_{t=1}^{T} H_t(b_t^*(\lambda^*)) = B \]

spend

The online loss function

\[ L(b_t^*, \lambda) = \sum_{t=1}^{T} L_t(b_t^*, \lambda) := \left[ v_t G_t(b_t^*) - \lambda H_t(b_t^*) \right] + \lambda B / T. \]

Derivative of the online loss

\[ L'_t(b_t^*, \lambda) = B / T - H_t(b_t^*) \]

discrepancy between forecasted and actual spend

Follow-the-leader: Find \( \lambda_{t+1} = \arg \min_{\lambda \geq 0} \sum_{\tau=1}^{t} L_\tau(b_\tau^*, \lambda). \)

Follow-the-regularized-leader: Online Gradient Descent, Online Mirror Descent, etc.
Other Types of Constraints

**Cost Control (ROI target)**

\[
\max_{b_t} \sum_{t=1}^{T} v_t G_t(b_t), \quad \text{s.t.} \quad \sum_{t=1}^{T} H_t(b_t) \leq \min \left( B, C \sum_{t=1}^{T} v_t G_t(b_t) \right).
\]

\[
b^*_t(\lambda, \mu) = \left( \frac{h_t}{g_t} \right)^{-1} \left( \frac{1 + \mu C'}{\lambda + \mu} \cdot v_t \right).
\]

**Budget Delivery Control**

\[
\max_{b_t} \sum_{t=1}^{T} v_t G_t(b_t), \quad \text{s.t.} \quad \sum_{t=1}^{T} H_t(b_t) \leq B, \sum_{t \in T_k} H_t(b_t) \leq B_k, \forall k.
\]

\[
b^*_t(\lambda, \lambda_k) = \left( \frac{h_t}{g_t} \right)^{-1} \left( \frac{v_t}{\lambda + \lambda_k} \right).
\]

**Guaranteed Delivery**

\[
\max_{b_t} \sum_{t=1}^{T} v_t G_t(b_t), \quad \text{s.t.} \quad \sum_{t=1}^{T} H_t(b_t) \leq B, \sum_{t \in T_k} v_t G_t(b_t) \geq V_k, \forall k.
\]

\[
b^*_t(\lambda, \mu_k) = \left( \frac{h_t}{g_t} \right)^{-1} \left( \frac{1 + \mu_k}{\lambda} \cdot v_t \right).
\]
Multiple Placements, Group of Ads and Marginal ROI

The proposed bidding framework ensures optimality, which implies equality of marginal ROI among all ads across all placements.
Engineering Considerations

- Batch online update
- Parameter Tuning
  - Normalization of $\lambda$
  - Empirical vs. Asymptotic Regret
- Forecast Error
  - Sensitivity Analysis on $T$
  - Relative Forecast vs. Total Forecast
- Model Predictive Control
Theorem (Gao, Yang, Chen, Liu, El Karoui ’22)

Competitor’s bids $\sim \text{Lognormal}(\mu, \sigma^2)$

value $\sim \text{Lognormal}(\mu', (\sigma')^2)$

Then optimal $\lambda^*$ is the solution to

$$e^{\mu + \frac{\sigma^2}{2}} \Phi \left( \frac{\mu' - \mu - \ln \lambda^* - \sigma^2}{\sqrt{(\sigma')^2 + \sigma^2}} \right) = \frac{B}{T}$$
Thank you!