Optimizing hierarchical queries for the attribution reporting API

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The Attribution Reporting API

- Privacy-preserving tool for ad conversion measurement on Chrome/Android
- Can produce aggregate statistics about conversion attribution without using persistent cross-site identifiers
- Summary reports satisfy differential privacy: noise is added to limit how much can be inferred about individual impressions

Conversion reporting

- Goal: estimate the number of conversions attributed to impressions, where the impressions and conversions have a certain combination of features
- E.g. how many conversions were attributed to impressions from campaign 123 and took place in Los Angeles last Friday?

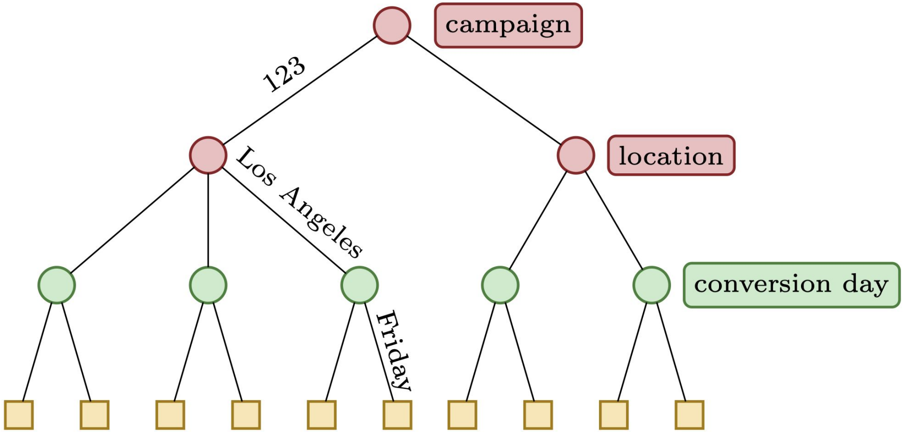
Click	campaign	location	conversion day
1	123	Paris	Monday
2	456	Chicago	Friday
3	789	London	*
4	123	Los Angeles	Friday
…	…	…	…

Hierarchical queries

- For each (city, day) of the campaign, how many attributed conversions?
- Higher-level aggregates: what is the total number of attributed conversions for this campaign? What about the total number in Los Angeles?
- Goal: given a tree that branches on impression/conversion features, want to estimate the number of conversions corresponding to each node in the tree.

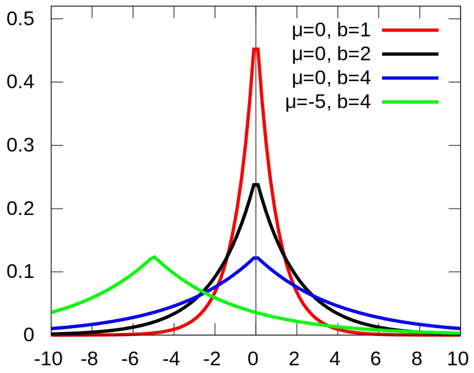
Error metric: thresholded RMS relative error averaged over the levels of the tree (same algorithms work with other metrics)

$$\mathrm{RMSRE}_\tau(T) := \sqrt{\mathbb{E}\left[\frac{1}{d+1}\sum_{i=0}^{d}\frac{1}{|L_i|}\sum_{v\in L_i}\left(\frac{|\hat{c}_v - c_v|}{\max(\tau, c)}\right)^2\right]}$$



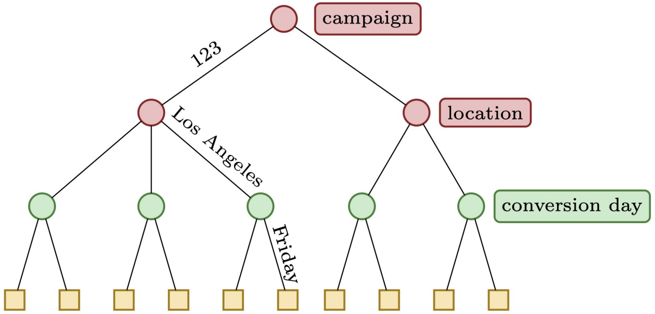
Differential privacy (DP) and the Laplace mechanism

- DP provides worst-case guarantees about how much an adversary can infer about a single row of the dataset
- Privacy level is controlled by a parameter ε > 0; smaller ε ⇔ more private
- For a counting query, can satisfy ε-DP by adding noise of scale 1/ε from a (continuous or discrete) Laplace distribution.
- Such estimates can be obtained using the Attribution Reporting API



Privacy budgeting and hierarchical queries

- What if multiple queries involve the same data record?
- Composition: Algorithms A_1, A_2 are ε_1-DP and ε_2-DP ⇒ (A_1,A_2) is $(\varepsilon_1+\varepsilon_2)$-DP
- In a tree:
  - Queries to different nodes at the same level touch disjoint subsets of the data
  - Queries to nodes at different levels may touch the same data record
- Given a total privacy budget ε, can allocate it to the d+1 levels of the tree so that $\varepsilon_0 + \varepsilon_1 + \cdots + \varepsilon_d = \varepsilon$



Main question

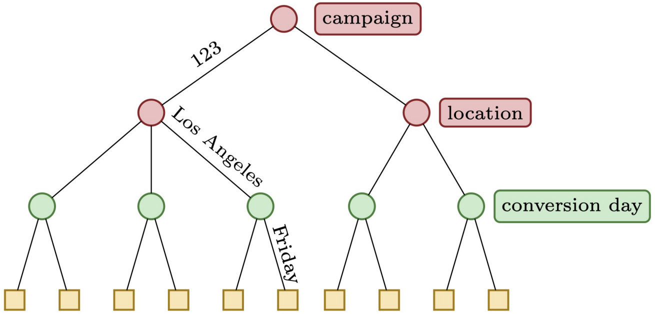
How can we obtain estimates for hierarchical queries that are consistent and have minimum possible error?

Two main results:

- A post-processing algorithm that reduces the error of estimates and ensures consistency with the hierarchical structure
- A procedure for optimizing the allocation of the privacy budget among the levels of the hierarchy

Post-processing algorithm

- Observation: the value of any internal node should equal the sum of the values of its children (*consistency*).
- Given independent estimates e_1, e_2 of the same quantity with variances v_1, v_2:
  - Can obtain other unbiased estimates by taking a convex combination $\alpha e_1 + (1- \alpha) e_2$
  - The optimal combination has $\alpha = v_2 / (v_1 + v_2)$, yielding an improved variance of $v_1 v_2/(v_1+v_2)$
- How can we optimally take into account all constraints encoded in the tree?



Post-processing algorithm

Given: estimates z_v of the count at each node v, and their variances var_v

Bottom-up pass. For each internal node v from largest to smallest depth:

Update z_v to be the minimum-variance convex combination of z_v and $\sum_{u \in child(v)} z_u$, and compute the corresponding variance var_v.

Top-down pass. For each internal node v from smallest to largest depth:

Update z_u for each $u \in$ child(v) by splitting the discrepancy $z_v - \sum_{u \in child(v)} z_u$ among the children proportionally to the variance var_u of each child estimate.

Output the final estimates z_u

Post-processing algorithm

- Optimal: computes best linear unbiased estimator
- Better privacy/accuracy tradeoff: given noisy estimates for each tree node, produces estimates with lower error, without any additional privacy leakage
- Produces consistent estimates
- Linear-time algorithm
- Can be extended to compute variances as well as estimates
- Extends the methods of [Hay et al., VLDB’10, Cormode et al., ICDE’12], which apply to regular trees; also related to the matrix mechanism of [Li et al., VLDB ’15, Nikolov et al., STOC’13], which in general requires ≥ quadratic time (n^{ω}).

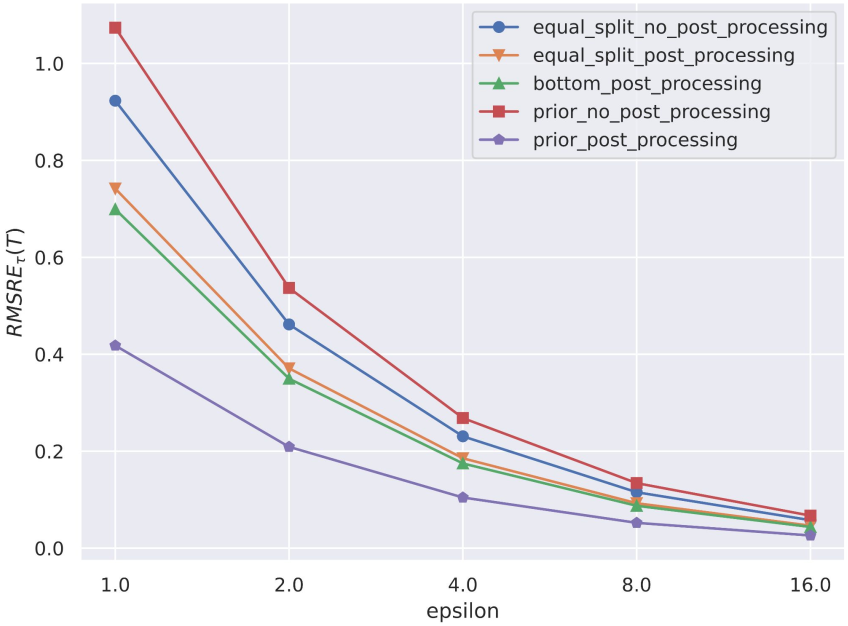
Allocating the privacy budget

- Post-processing tells us the optimal way to use a set of measurements, but which measurements should we take?
- For total budget ε, can split it in many ways among the levels of the tree
- Given (noisy) historical data or a prior, can compare these options
- Optimize to choose the best privacy budget split
- Simple greedy approach:
    - Divide total budget into k increments
    - In each iteration, allocate ε/k additional budget to the level that most decreases the overall error after post-processing

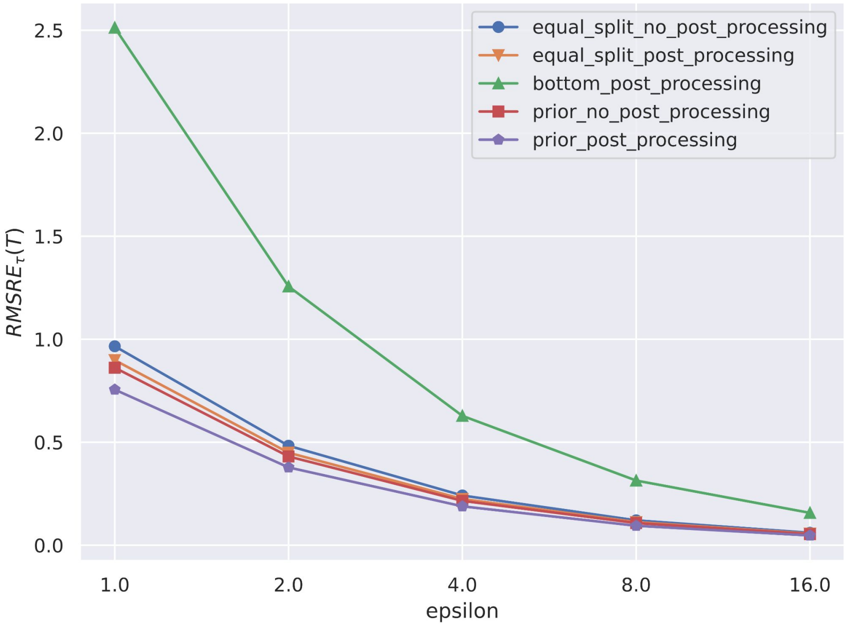
Evaluation

- Evaluated on two public Criteo datasets, Sponsored Search Conversion Log (CSSCL) and Attribute Modeling for Bidding (CAMB)
- Selected attributes from each dataset to construct hierarchy
- Split datasets into budgeting data and test data based on click time
- Compared five approaches:
  - equal budget split, with and without post-processing
  - all budget on bottom level, with post-processing
  - optimizing per-level privacy budgets, with and without post-processing

Evaluation

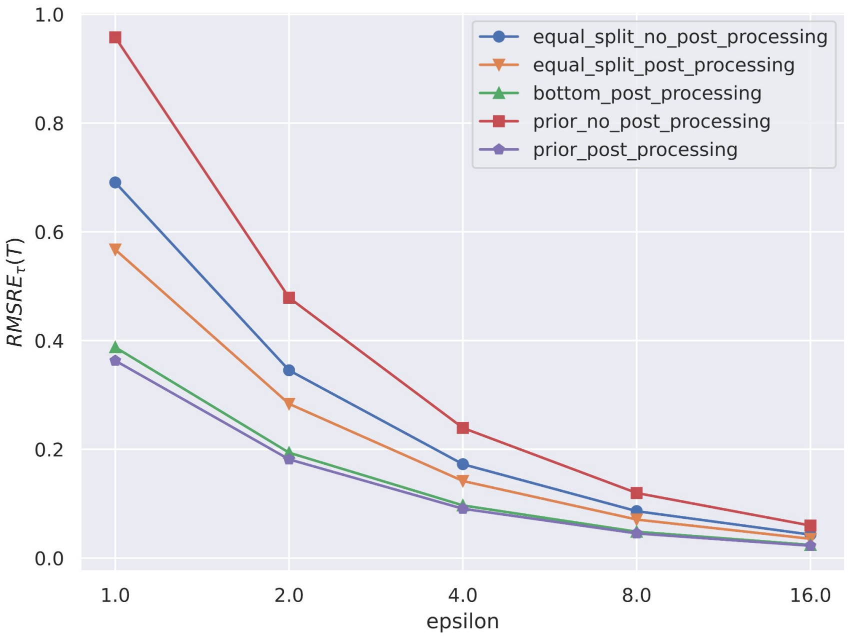


Five-attribute hierarchy using Criteo Sponsored Search Conversion Log (CSSCL) dataset, τ = 10

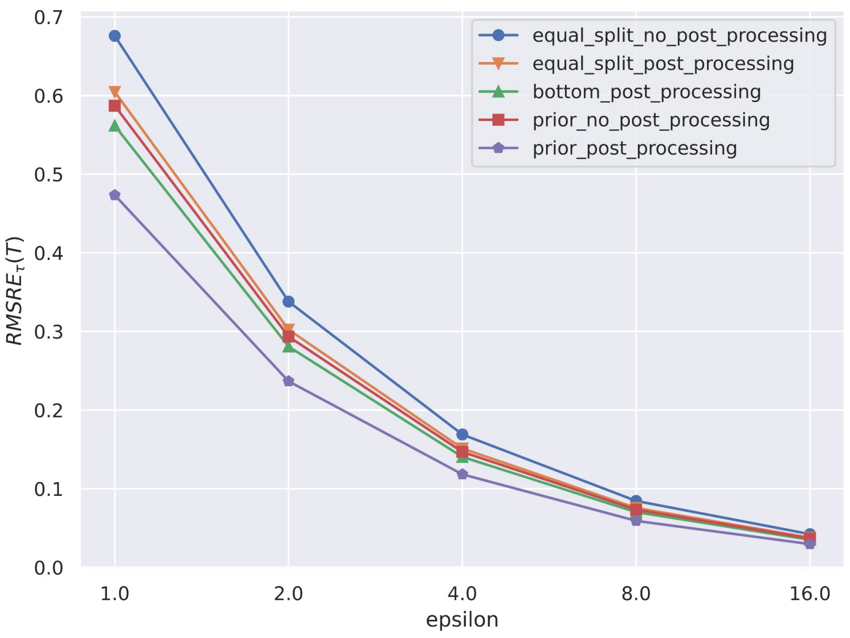


Four-attribute hierarchy using Criteo Attribution Modeling for Bidding (CAMB) dataset, τ = 10

Evaluation



Four-attribute hierarchy using Criteo Sponsored Search Conversion Log (CSSCL) dataset, τ = 10



Three-attribute hierarchy using Criteo Attribution Modeling for Bidding (CAMB) dataset, τ = 10